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Multiple-set split feasibility problems for κ-asymptotically strictly pseudo-nonspreading mappings in Hilbert spaces

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Abstract

Some weak and strong convergence theorems for solving multiple-set split feasibility problems for κ-asymptotically strictly pseudo-nonspreading mappings in infinite-dimensional Hilbert spaces are proved. The results presented in the paper extend and improve the corresponding results of Xu (Inverse Probl. 22(6):2021-2034, 2006), Osilike and Isiogugu (Nonlinear Anal. 74:1814-1822, 2011), Chang *et al.* (Abstr. Appl. Anal. 2012:491760, 2012), and others.

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1 Introduction

Throughout this article, we always assume that H_1, H_2 are real Hilbert spaces; '$\to$' and '$\rightharpoonup$' denote strong and weak convergence, respectively.

The split feasibility problem (*SFP*) in finite dimensional spaces was first introduced by Censor and Elfving [1] for modeling inverse problems. The (*SFP*) can be used in various disciplines such as medical image reconstruction [2], image restoration, computer tomography, and radiation therapy treatment planning [3–5]. The multiple-set split feasibility problem (*MSSFP*) was studied in [4–7].

Let $A : H_1 \to H_2$ be a bounded linear operator, $S_i : H_1 \to H_1$ and $T_i : H_2 \to H_2$, $i = 1, 2, \ldots, N$, be two finite families of mappings, $C := \bigcap_{i=1}^N F(S_i)$ and $Q := \bigcap_{i=1}^N F(T_i)$, where $F(S_i)$ and $F(T_i)$ are the sets of fixed points of S_i and T_i, respectively.

The so-called *multiple set split feasibility problem* is

$$\text{to find } x^* \in C \text{ such that } Ax^* \in Q. \tag{1.1}$$

In the sequel, we use Γ to denote the set of solutions of the problem (*MSSFP*) (1.1), that is,

$$\Gamma = \{x \in C : Ax \in Q\}. \tag{1.2}$$

Let H be a real Hilbert space and K be a nonempty closed convex subset of H. Following Kohsaka and Takahashi [8–11], a mapping $T: K \to K$ is said to be *nonspreading* if

$$2\|Tx - Ty\|^2 \le \|Tx - y\|^2 + \|Ty - x\|^2 \quad \text{for all } x, y \in K.$$

It is to see that the above inequality is equivalent to

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + 2\langle x - Tx, y - Ty\rangle \quad \text{for all } x, y \in K.$$

In 1967, Browder and Petryshyn [12] introduced the concept of κ-strictly pseudo-nonspreading mapping.

Definition 1.1 [12] Let H be a real Hilbert space. A mapping $T: D(T) \subset H \to H$ is said to be *κ-strictly pseudo-nonspreading* if there exists $\kappa \in [0,1)$ such that

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + \kappa \big\|x - Tx - (y - Ty)\big\|^2 + 2\langle x - Tx, y - Ty\rangle, \quad \forall x, y \in D(T).$$

Clearly, every nonspreading mapping is κ-strictly pseudo-nonspreading.

The class of asymptotically strict pseudo-contractions was introduced by Qihou [13] in 1996. Kim and Xu [14], Inchan and Nammanee [15], Zhou [16] Cho [17], and Ge [18] proved that the class of asymptotically strict pseudo-contractions is demiclosed at the origin and also obtained some weak convergence theorems for the class of mappings. In 2012, Osilike and Isiogugu [19] introduced a class of *nonspreading type mappings* which is more general than the class studied in [11] in Hilbert spaces and proved some weak and strong convergence theorems in real Hilbert spaces. Recently, Chang *et al.* [7] studied the multiple-set split feasibility problem for an asymptotically strict pseudo-contraction in the framework of infinite-dimensional Hilbert spaces.

Definition 1.2 [7] Let H be a real Hilbert space, we say that the mapping $T: D(T) \subset H \to H$ is a κ-asymptotically strict pseudo-contraction if there exists a constant $\kappa \in [0,1)$ and a sequence $\{k_n\} \subset [1,\infty)$ with $k_n \to 1$ $(n \to \infty)$ such that

$$\big\|T^n x - T^n y\big\|^2 \le k_n \|x - y\|^2 + \kappa \big\|x - T^n x - \big(y - T^n y\big)\big\|^2$$

holds for all $x, y \in D(T)$.

In this article we introduce the following class of *κ-asymptotically strictly pseudo-nonspreading* mappings which is more general than that of κ-strictly pseudo-nonspreading mappings and κ-asymptotically strict pseudo-contractions.

Definition 1.3 Let H be a real Hilbert space. A mapping $T: D(T) \subset H \to H$ is said to be *κ-asymptotically strictly pseudo-nonspreading* if there exists a constant $\kappa \in [0,1)$ and a sequence $\{k_n\} \subset [1,\infty)$ with $k_n \to 1$ $(n \to \infty)$ such that

$$\begin{aligned} &\big\|T^n x - T^n y\big\|^2 \le k_n \|x - y\|^2 + \kappa \big\|x - T^n x - \big(y - T^n y\big)\big\|^2 + 2\big\langle x - T^n x, y - T^n y\big\rangle, \\ &\quad \forall x, y \in D(T). \end{aligned} \tag{1.3}$$

Example 1.4 Now, we give an example of κ-asymptotically strict pseudo-contractive mapping.

Let C be a unit ball in a real Hilbert l^2, and let $T: C \to C$ be a mapping defined by

$$T: \quad (x_1, x_2, \ldots) \to (0, x_1^2, a_2 x_2, a_3 x_3, \ldots), \tag{1.4}$$

where $\{a_i\}$ is a sequence in $(0,1)$ such that $\prod_{i=2}^{\infty} \alpha_i = \frac{1}{2}$.

It is proved in Goebel and Kirk [20] that

(i) $\|Tx - Ty\| \leq 2\|x - y\|$, $\forall x, y \in C$;

(ii) $\|T^n x - T^n y\| \leq 2\prod_{i=2}^{n} a_j \|x - y\|$, $\forall n \geq 2$ and $x, y \in C$.

Define $k_1^{\frac{1}{2}} = 2$, $k_n^{\frac{1}{2}} = 2\prod_{i=2}^{n} a_j$, $n \geq 2$, then

$$\lim_{n\to\infty} k_n = \lim_{n\to\infty} \left(2\prod_{i=2}^{n} a_j\right)^2 = 1.$$

Letting $\kappa = 0$, then $\forall x, y \in C$, $n \geq 1$, we have

$$\underline{\left\|T^n x - T^n y\right\|^2 \leq k_n \|x - y\|^2 + \kappa \left\|x - y - \left(T^n x - T^n y\right)\right\|^2.}$$

This implies that T is a κ-asymptotically strict pseudo-contractive mapping.

Example 1.5 Now, we give an example of κ-asymptotically strictly pseudo-nonspreading mapping.

Let $X = l^2$ with the norm $\|\cdot\|$ be defined by

$$\|x\| = \sqrt{\sum_{i=1}^{\infty} x_i^2}, \quad \forall x = (x_1, x_2, \ldots, x_n, \ldots) \in X,$$

and let $C = \{x = (x_1, x_2, \ldots, x_n, \ldots) | x_i \in R^1, i = 1, 2, \ldots\}$ be an orthogonal subspace of X (*i.e.*, $\forall x, y \in C$, we have $\langle x, y\rangle = 0$). It is obvious that C is a nonempty closed convex subset of X. For each $x = (x_1, x_2, \ldots, x_n, \ldots) \in C$, we define a mapping $T: C \to C$ by

$$Tx = \begin{cases} (x_1, x_2, \ldots, x_n, \ldots) & \text{if } \prod_{i=1}^{\infty} x_i < 0; \\ (-x_1, -x_2, \ldots, -x_n, \ldots) & \text{if } \prod_{i=1}^{\infty} x_i \geq 0. \end{cases} \tag{1.5}$$

Next we prove that T is a κ-asymptotically strictly pseudo-nonspreading mapping.

In fact, for any $x, y \in C$, we have the following cases.

Case 1. If $\prod_{i=1}^{\infty} x_i < 0$ and $\prod_{i=1}^{\infty} y_i < 0$, then we have $T^n x = x$, $T^n y = y$, and so then inequality (1.3) holds.

Case 2. If $\prod_{i=1}^{\infty} x_i < 0$ and $\prod_{i=1}^{\infty} y_i \geq 0$, then we have that $T^n x = x$, $T^n y = (-1)^n y$. This implies that

$$\begin{cases} \|T^n x - T^n y\|^2 = \|x - (-1)^n y\|^2 = \|x\|^2 + \|y\|^2; \\ k_n \|x - y\|^2 = k_n(\|x\|^2 + \|y\|^2); \\ \|x - T^n x - (y - T^n y)\|^2 = [1 - (-1)^n]^2 \|y\|^2; \\ 2\langle x - T^n x, y - T^n y\rangle = 0. \end{cases}$$

Therefore inequality (1.3) holds.

Case 3. If $\prod_{i=1}^{\infty} x_i \geq 0$ and $\prod_{i=1}^{\infty} y_i \geq 0$, then we have $T^n x = (-1)^n x$, $T^n y = (-1)^n y$. Hence we have

$$\begin{cases} \|T^n x - T^n y\|^2 = \|(-1)^n x - (-1)^n y\|^2 = \|x-y\|^2 = \|x\|^2 + \|y\|^2; \\ k_n\|x-y\|^2 = k_n(\|x\|^2 + \|y\|^2); \\ \|x - T^n x - (y - T^n y)\|^2 = [1-(-1)^n]^2\|x-y\|^2 = [1-(-1)^n]^2(\|x\|^2 + \|y\|^2); \\ 2\langle x - T^n x, y - T^n y\rangle = 0. \end{cases}$$

Thus inequality (1.3) still holds. Therefore the mapping defined by (1.5) is a κ-asymptotically strictly pseudo-nonspreading mapping.

The purpose of this article is under suitable conditions to prove some weak and strong convergence theorems for solving multiple-set split feasibility problem (1.1) for a κ-asymptotically strictly pseudo-nonspreading mapping in infinite-dimensional Hilbert spaces. The results presented in the paper extend and improve the corresponding results of Xu [6], Osilike and Isiogugu [19], Chang *et al.* [7], and many others.

2 Preliminaries

In the sequel, we first recall some definitions, notations, and conclusions which will be needed in proving our main results.

Let E be a real Banach space. A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be *demiclosed* at origin if whenever $\{x_n\}$ is a sequence in $D(T)$ converging weakly to a point $x^* \in D(T)$ and $\|(I-T)x_n\|$ converging strongly to 0, then $Tx^* = x^*$.

A Banach space E is said to have the *Opial* property if, for any sequence $\{x_n\}$ with $x_n \rightharpoonup x^*$, we have

$$\liminf_{n\to\infty}\|x_n - x^*\| < \liminf_{n\to\infty}\|x_n - y\|$$

for all $y \in E$ with $y \neq x^*$.

It is well known that each Hilbert space possesses the Opial property.

A mapping $T: K \to K$ is said to be *semicompact* if for any bounded sequence $\{x_n\} \subset K$ with $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$, there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges strongly to some point $x^* \in K$.

A mapping $T: K \to K$ is said to be *uniformly L-Lipschitzian* if there exists a constant $L > 0$ such that

$$\|T^n x - T^n y\| \leq L\|x-y\|, \quad \forall x, y \in K.$$

Let K be a nonempty closed convex subset of a real Hilbert space H. The *metric projection* $P_K: H \to K$ is a mapping such that for each $x \in H$, $P_K x$ is the unique point in K such that $\|x - P_K x\| \leq \|x - y\|$, $\forall y \in K$. It is known that for each $x \in H$,

$$\langle x - P_K x, y - P_K x\rangle \leq 0, \quad \forall y \in K.$$

Lemma 2.1 *Let H be a real Hilbert space, then the following results hold*:

(i) *For all* $x, y \in H$ *and for all* $t \in [0,1]$,

$$\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2.$$

(ii) $\|x+y\|^2 \le \|x\|^2 + 2\langle y, x+y\rangle$.

(iii) *If* $\{x_n\}_{n=1}^\infty$ *is a sequence in* H *which converges weakly to* $z \in H$, *then*

$$\limsup_{n\to\infty} \|x_n - y\|^2 = \limsup_{n\to\infty} \|x_n - z\|^2 + \|z-y\|^2, \quad \forall y \in H.$$

Lemma 2.2 *Let* K *be a nonempty closed convex subset of a real Hilbert space* H, *and let* $T : K \to K$ *be a continuous* κ*-asymptotically strictly pseudo-nonspreading mapping. If* $F(T) \neq \emptyset$, *then it is a closed and convex subset.*

Proof Let $\{x_n\} \subset F(T)$ be a sequence such that $\lim_{n\to\infty} x_n = x^* \in K$. Now we prove that $x^* \in F(T)$. In fact, since T is κ-asymptotically strictly pseudo-nonspreading, for each $m \ge 1$, we have

$$\begin{aligned}\|T^m x^* - x_n\|^2 &= \|T^m x^* - T^m x_n\|^2 \\ &\le k_m \|x_n - x^*\|^2 + 2\langle x^* - T^m x^*, x_n - T^m x_n\rangle \\ &\quad + \kappa \|x^* - T^m x^* - (x_n - T^m x_n)\|^2 \\ &= k_m \|x_n - x^*\|^2 + \kappa \|x^* - T^m x^*\|^2.\end{aligned}$$

Taking the limit as $n \to \infty$ in the above inequality, we have

$$\|T^m x^* - x^*\|^2 \le \kappa \|x^* - T^m x^*\|^2.$$

Since $\kappa \in (0,1)$, we have $\|T^m x^* - x^*\| = 0$ for each $m \ge 1$. Hence $Tx^* = x^*$. This shows that $F(T)$ is closed.

Now we prove that $F(T)$ is convex. In fact, let $p_1, p_2 \in F(T)$, and $z = \lambda p_1 + (1-\lambda)p_2$, we prove that $z \in F(T)$. Since $p_1 - z = (1-\lambda)(p_1 - p_2)$ and $p_2 - z = \lambda(p_2 - p_1)$, by using Lemma 2.1(i), we have

$$\begin{aligned}\|z - T^m z\|^2 &= \|\lambda(p_1 - T^m z) + (1-\lambda)(p_2 - T^m z)\|^2 \\ &= \lambda\|p_1 - T^m z\|^2 + (1-\lambda)\|p_2 - T^m z\|^2 - \lambda(1-\lambda)\|p_1 - p_2\|^2 \\ &\le \lambda\big(k_m\|p_1 - z\|^2 + \kappa\|p_1 - T^m p_1 - (z - T^m z)\|^2 + 2\langle p_1 - T^m p_1, z - T^m z\rangle\big) \\ &\quad + (1-\lambda)\big(k_m\|p_2 - z\|^2 + \kappa\|p_2 - T^m p_2 - (z - T^m z)\|^2 \\ &\quad + 2\langle p_2 - T^m p_2, z - T^m z\rangle\big) - \lambda(1-\lambda)\|p_1 - p_2\|^2 \\ &= \lambda\big(k_m\|p_1 - z\|^2 + \kappa\|z - T^m z\|^2\big) + (1-\lambda)\big(k_m\|p_2 - z\|^2 + \kappa\|z - T^m z\|^2\big) \\ &\quad - \lambda(1-\lambda)\|p_1 - p_2\|^2.\end{aligned}$$

Taking $\limsup_{m\to\infty}$ on both sides of the above inequality, we have

$$\limsup_{m\to\infty}\|z - T^m z\|^2 \le \limsup_{m\to\infty} \kappa\|z - T^m z\|^2.$$

Since $\kappa < 1$, we have

$$\limsup_{m\to\infty} \left\| T^m z - z \right\|^2 = 0,$$

and so $\lim_{m\to\infty} T^m z = z$, *i.e.*, $Tz = z$. This completes the proof. □

Lemma 2.3 *Let K be a nonempty closed convex subset of a real Hilbert space H, and let $T : K \to K$ be a continuous κ-asymptotically strictly pseudo-nonspreading mapping. Then $(I - T)$ is demiclosed at 0, that is, if $x_n \rightharpoonup x^*$ and $\limsup_{m\to\infty} \limsup_{n\to\infty} \|(I - T^m)x_n\| = 0$, then $\|(I - T)x^*\| = 0$.*

Proof Since $\{x_n\}$ is weak convergence, $\{x_n\}$ is bounded. For each $x \in H$, define $f : H \to [0,\infty)$ by

$$f(x) := \limsup_{n\to\infty} \|x_n - x\|^2, \quad x \in H.$$

From Lemma 2.1(iii), we have

$$f(x) = \limsup_{n\to\infty} \left\| x_n - x^* \right\|^2 + \left\| x^* - x \right\|^2, \quad x \in H.$$

Thus we have

$$f(x) = f\left(x^*\right) + \left\| x - x^* \right\|^2, \quad x \in H.$$

In particular, for each $m \geq 1$,

$$f\left(T^m x^*\right) = f\left(x^*\right) + \left\| T^m x^* - x^* \right\|^2. \tag{2.1}$$

On the other hand, we have

$$\begin{aligned} f\left(T^m x^*\right) &= \limsup_{n\to\infty} \left\| x_n - T^m x^* \right\|^2 \\ &= \limsup_{n\to\infty} \left\| x_n - T^m x_n + T^m x_n - T^m x^* \right\|^2 \\ &= \limsup_{n\to\infty} \left(\left\| x_n - T^m x_n \right\|^2 + 2\left\langle x_n - T^m x_n, T^m x_n - T^m x^* \right\rangle + \left\| T^m x_n - T^m x^* \right\|^2 \right). \end{aligned}$$

Since $\limsup_{m\to\infty} \limsup_{n\to\infty} \|(I - T^m)x_n\| = 0$ and T is a κ-asymptotically strictly pseudo-nonspreading mapping, taking $\limsup_{m\to\infty}$ on both sides of the above equality, we get

$$\begin{aligned} \limsup_{m\to\infty} f\left(T^m x^*\right) &\leq \limsup_{m\to\infty} \left\| T^m x_n - T^m x^* \right\|^2 \\ &\leq \limsup_{m\to\infty} \limsup_{n\to\infty} \Big(k_m \left\| x_n - x^* \right\|^2 + \kappa \left\| x_n - T^m x_n - \left(x^* - T^m x^* \right) \right\|^2 \\ &\quad + 2\left\langle x_n - T^m x_n, x^* - T^m x^* \right\rangle \Big). \end{aligned}$$

By virtue of $\limsup_{m\to\infty} \limsup_{n\to\infty} \|(I - T^m)x_n\| = 0$ and $k_m \to 1$ $(m \to \infty)$, we have

$$\limsup_{m\to\infty} f\bigl(T^m x^*\bigr) \le f\bigl(x^*\bigr) + \limsup_{m\to\infty} \kappa \bigl\| x^* - T^m x^* \bigr\|^2. \tag{2.2}$$

On the other hand, it follows from (2.1) that

$$\limsup_{m\to\infty} f\bigl(T^m x^*\bigr) = f\bigl(x^*\bigr) + \limsup_{m\to\infty} \bigl\| T^m x^* - x^* \bigr\|^2, \quad \forall x \in H. \tag{2.3}$$

Since $\kappa < 1$, it follows from (2.2) and (2.3) that $\limsup_{m\to\infty} \|T^m x^* - x^*\|^2 = 0$. So $\lim_{m\to\infty} T^m x^* = x^*$ and $Tx^* = x^*$. This completes the proof. □

3 Main results

Theorem 3.1 *Let H_1, H_2, A, $\{S_i\}$, $\{T_i\}$, C, Q be the same as in multiple set split feasibility problem* (1.1). *For each $i = 1, 2, \ldots, N$, let T_i be a uniformly $\tilde{L}_i$-Lipschitzian and κ_i-asymptotically strictly pseudo-nonspreading mapping, S_i be a uniformly L_i-Lipschitzian and ϱ_i-asymptotically strictly pseudo-nonspreading mapping. Let $\{x_n\}$ be the sequence generated by*

$$\begin{cases} x_1 \in H_1 \text{ chosen arbitrarily}, \\ u_n = x_n + \gamma A^*(T^n_{n(\operatorname{mod} N)} - I)Ax_n, \\ x_{n+1} = (1 - \alpha_n)u_n + \alpha_n S^n_{n(\operatorname{mod} N)} u_n, \end{cases} \tag{3.1}$$

where γ is a constant and $\gamma \in (0, \frac{1-\kappa}{\lambda})$, λ is the spectral of the operator A^*A, $\kappa = \max\{\kappa_1, \kappa_2, \ldots, \kappa_N\}$ and $\{\alpha_n\}$ is a sequence in $(0, 1 - \varrho]$ with $\varrho = \max\{\varrho_1, \varrho_2, \ldots, \varrho_N\}$. If $\Gamma \neq \emptyset$, then the sequence $\{x_n\}$ converges weakly to a point $x^* \in \Gamma$.

Proof The proof is divided into five steps.

(I) We first prove the limit $\lim_{n\to\infty} \|x_n - p\|$ exists for any $p \in \Gamma$.

Since $p \in \Gamma$, we have $p \in C := \bigcap_{i=1}^N F(S_i)$ and $Ap \in Q := \bigcap_{i=1}^N F(T_i)$. It follows from (3.1) that

$$\begin{aligned} \|x_{n+1} - p\|^2 &= \bigl\| u_n - p + \alpha_n \bigl(S^n_{n(\operatorname{mod} N)} u_n - u_n\bigr) \bigr\|^2 \\ &= \|u_n - p\|^2 + 2\alpha_n \bigl\langle u_n - p, S^n_{n(\operatorname{mod} N)} u_n - u_n \bigr\rangle \\ &\quad + \alpha_n^2 \bigl\| u_n - S^n_{n(\operatorname{mod} N)} u_n \bigr\|^2. \end{aligned} \tag{3.2}$$

Because S_i is a ϱ_i-asymptotically strictly pseudo-nonspreading mapping, for any $v \in H_1$, we have

$$\begin{aligned} &\bigl\| S^n_{n(\operatorname{mod} N)} u_n - S^n_{n(\operatorname{mod} N)} v \bigr\|^2 \\ &\quad \le \|u_n - v\|^2 + \varrho \bigl\| u_n - S^n_{n(\operatorname{mod} N)} u_n - \bigl(v - S^n_{n(\operatorname{mod} N)} v\bigr) \bigr\|^2 \\ &\qquad + 2\bigl\langle u_n - S^n_{n(\operatorname{mod} N)} u_n, v - S^n_{n(\operatorname{mod} N)} v \bigr\rangle. \end{aligned}$$

Taking $v = p$, we have

$$\bigl\| S^n_{n(\operatorname{mod} N)} u_n - p \bigr\|^2 \le \|u_n - p\|^2 + \varrho \bigl\| u_n - S^n_{n(\operatorname{mod} N)} u_n \bigr\|^2.$$

Therefore we have

$$\begin{aligned}\|S^n_{n(\bmod N)}u_n - p\|^2 &= \|S^n_{n(\bmod N)}u_n - u_n + u_n - p\|^2 \\ &= \|S^n_{n(\bmod N)}u_n - u_n\|^2 + 2\langle S^n_{n(\bmod N)}u_n - u_n, u_n - p\rangle + \|u_n - p\|^2 \\ &\le \|u_n - p\|^2 + \varrho\|u_n - S^n_{n(\bmod N)}u_n\|^2.\end{aligned}$$

Simplifying the above inequality, we have that

$$2\alpha_n\langle S^n_{n(\bmod N)}u_n - u_n, u_n - p\rangle \le \alpha_n(\varrho - 1)\|u_n - S^n_{n(\bmod N)}u_n\|^2. \tag{3.3}$$

It follows from (3.2) and (3.3) that

$$\begin{aligned}&\|x_{n+1} - p\|^2 \\ &\quad\le \|u_n - p\|^2 + \alpha_n(\varrho - 1)\|u_n - S^n_{n(\bmod N)}u_n\|^2 + \alpha_n^2\|u_n - S^n_{n(\bmod N)}u_n\|^2 \\ &\quad= \|u_n - p\|^2 - \alpha_n(1 - \varrho - \alpha_n)\|u_n - S^n_{n(\bmod N)}u_n\|^2.\end{aligned} \tag{3.4}$$

On the other hand,

$$\begin{aligned}\|u_n - p\|^2 &= \|x_n - p + \gamma A^*(T^n_{n(\bmod N)} - I)Ax_n\|^2 \\ &= \|x_n - p\|^2 + 2\gamma\langle x_n - p, A^*(T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &\quad + \gamma^2\|A^*(T^n_{n(\bmod N)} - I)Ax_n\|^2 \\ &= \|x_n - p\|^2 + 2\gamma\langle x_n - p, A^*(T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &\quad + \gamma^2\langle A^*(T^n_{n(\bmod N)} - I)Ax_n, A^*(T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &= \|x_n - p\|^2 + 2\gamma\langle x_n - p, A^*(T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &\quad + \gamma^2\langle AA^*(T^n_{n(\bmod N)} - I)Ax_n, (T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &\le \|x_n - p\|^2 + 2\gamma\langle x_n - p, A^*(T^n_{n(\bmod N)} - I)Ax_n\rangle \\ &\quad + \gamma^2\|A\|^2\|(T^n_{n(\bmod N)} - I)Ax_n\|^2.\end{aligned} \tag{3.5}$$

Since T_i is a κ_i-asymptotically strictly pseudo-nonspreading mapping and noting $Ap \in \bigcap_{i-1}^N F(T_i)$, we have

$$\begin{aligned}\|T^n_{n(\bmod N)}Ax_n - Ap\|^2 &= \|T^n_{n(\bmod N)}Ax_n - T^n_{n(\bmod N)}Ap\|^2 \\ &\le \|Ax_n - Ap\|^2 + \kappa\|T^n_{n(\bmod N)}Ax_n - Ax_n\|^2.\end{aligned} \tag{3.6}$$

Again since

$$\begin{aligned}\|T^n_{n(\bmod N)}Ax_n - Ap\|^2 &= \|T^n_{n(\bmod N)}Ax_n - Ax_n + Ax_n - Ap\|^2 \\ &= \|T^n_{n(\bmod N)}Ax_n - Ax_n\|^2 + \|Ax_n - Ap\|^2 \\ &\quad + 2\langle T^n_{n(\bmod N)}Ax_n - Ax_n, Ax_n - Ap\rangle,\end{aligned} \tag{3.7}$$

hence from (3.6) and (3.7) we have that

$$2\langle T^n_{n(\operatorname{mod} N)}Ax_n - Ax_n, Ax_n - Ap\rangle \le (\kappa - 1)\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2. \tag{3.8}$$

By virtue of (3.8) we have

$$\begin{aligned}
&\langle T^n_{n(\operatorname{mod} N)}Ax_n - Ax_n, T^n_{n(\operatorname{mod} N)}Ax_n - Ap\rangle \\
&\quad = \langle T^n_{n(\operatorname{mod} N)}Ax_n - Ax_n, T^n_{n(\operatorname{mod} N)}Ax_n - Ap + Ax_n - Ax_n\rangle \\
&\quad = \big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 + \langle T^n_{n(\operatorname{mod} N)}Ax_n - Ax_n, Ax_n - Ap\rangle \\
&\quad \le \big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 + \frac{\kappa - 1}{2}\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 \\
&\quad = \frac{\kappa + 1}{2}\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2.
\end{aligned} \tag{3.9}$$

It follows from (3.9) that

$$\begin{aligned}
&2\gamma\langle x_n - p, A^*\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\rangle \\
&\quad = 2\gamma\langle A(x_n - p), \big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\rangle \\
&\quad = 2\gamma\langle A(x_n - p) + \big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n - \big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n, \big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\rangle \\
&\quad = 2\gamma\langle T^n_{n(\operatorname{mod} N)}Ax_n - Ap, \big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\rangle - 2\gamma\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 \\
&\quad \le \big[\gamma(1 + \kappa) - 2\gamma\big]\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 \\
&\quad = \gamma(\kappa - 1)\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2.
\end{aligned} \tag{3.10}$$

Substituting (3.10) into (3.5) and then substituting the resulting inequality into (3.4), we have

$$\begin{aligned}
&\|x_{n+1} - p\|^2 \\
&\quad \le \|x_n - p\|^2 + \gamma^2\|A\|^2\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 + \big[\gamma(\kappa - 1)\big]\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 \\
&\qquad - \alpha_n(1 - \kappa - \alpha_n)\big\|u_n - S^n_{n(\operatorname{mod} N)}u_n\big\|^2 \\
&\quad \le \|x_n - p\|^2 - \gamma\big(1 - \kappa - \gamma\|A\|^2\big)\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 \\
&\qquad - \alpha_n(1 - \kappa - \alpha_n)\big\|u_n - S^n_{n(\operatorname{mod} N)}u_n\big\|^2 \\
&\quad \le \|x_n - p\|^2.
\end{aligned} \tag{3.11}$$

This shows that the limit $\lim_{n\to\infty}\|x_n - p\|$ exists.

(II) Now we prove that the limit $\lim_{n\to\infty}\|u_n - p\|$ exists.

By (3.11) we have

$$\begin{aligned}
\gamma\big(1 - \kappa - \gamma\|A\|^2\big)\big\|\big(T^n_{n(\operatorname{mod} N)} - I\big)Ax_n\big\|^2 + \alpha_n(1 - \kappa - \alpha_n)\big\|u_n - S^n_{n(\operatorname{mod} N)}u_n\big\|^2 \\
\le \|x_n - p\|^2 - \|x_{n+1} - p\|^2.
\end{aligned}$$

This implies that

$$\lim_{n\to\infty}\left\|\left(T^n_{n(\bmod N)}-I\right)Ax_n\right\|=0, \tag{3.12}$$

and

$$\lim_{n\to\infty}\left\|u_n-S^n_{n(\bmod N)}u_n\right\|=0. \tag{3.13}$$

It follows from (3.5), (3.12), and (3.13) that the limit $\lim_{n\to\infty}\|u_n-p\|$ exists and

$$\lim_{n\to\infty}\|x_n-p\|=\lim_{n\to\infty}\|u_n-p\|.$$

(III) Now, we prove that $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$, $\lim_{n\to\infty}\|u_{n+1}-u_n\|=0$.
In fact, it follows from (3.1) that

$$\begin{aligned}
&\|x_{n+1}-x_n\|\\
&\quad=\left\|(1-\alpha_n)u_n+\alpha_nS^n_{n(\bmod N)}u_n-x_n\right\|\\
&\quad=\left\|(1-\alpha_n)\left(x_n+\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right)+\alpha_nS^n_{n(\bmod N)}u_n-x_n\right\|\\
&\quad=\left\|(1-\alpha_n)\left(\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right)+\alpha_n\left(S^n_{n(\bmod N)}u_n-x_n\right)\right\|\\
&\quad=\left\|(1-\alpha_n)\left(\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right)+\alpha_n\left(S^n_{n(\bmod N)}u_n-u_n\right)+\alpha_n(u_n-x_n)\right\|\\
&\quad=\left\|(1-\alpha_n)\left(\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right)+\alpha_n\left(S^n_{n(\bmod N)}u_n-u_n\right)\right.\\
&\qquad\left.+\alpha_n\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right\|\\
&\quad=\left\|\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n+\alpha_n\left(S^n_{n(\bmod N)}u_n-u_n\right)\right\|.
\end{aligned}\tag{3.14}$$

This together with (3.12) and (3.13) shows that

$$\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \tag{3.15}$$

Similarly, it follows from (3.1), (3.12), and (3.15) that

$$\begin{aligned}
&\|u_{n+1}-u_n\|\\
&\quad=\left\|x_{n+1}+\gamma A^*\left(T^{n+1}_{n+1(\bmod N)}-I\right)Ax_{n+1}-\left[x_n+\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right]\right\|\\
&\quad\le\|x_{n+1}-x_n\|+\left\|\gamma A^*\left(T^{n+1}_{n+1(\bmod N)}-I\right)Ax_{n+1}\right\|+\left\|\gamma A^*\left(T^n_{n(\bmod N)}-I\right)Ax_n\right\|\\
&\quad\to 0\quad(\text{as } n\to\infty).
\end{aligned}\tag{3.16}$$

(IV) We prove that, for each $j=1,2,\ldots,N$,

$$\|u_{iN+j}-S_ju_{iN+j}\|\to 0,\qquad \|Ax_{iN+j}-T_jAx_{iN+j}\|\to 0\quad(i\to\infty). \tag{3.17}$$

In fact, it follows from (3.13) that

$$\left\|u_{iN+j}-S_j^{iN+j}u_{iN+j}\right\|\to 0\quad(i\to\infty). \tag{3.18}$$

Since S_j is uniformly L_j-Lipschitzian continuous, it follows from (3.16) and (3.18) that

$$\begin{aligned}
&\|u_{iN+j} - S_j u_{iN+j}\| \\
&\quad \le \|u_{iN+j} - S_j^{iN+j} u_{iN+j}\| + \|S_j^{iN+j} u_{iN+j} - S_j u_{iN+j}\| \\
&\quad \le \|u_{iN+j} - S_j^{iN+j} u_{iN+j}\| + L_j \|S_j^{iN+j-1} u_{iN+j} - u_{iN+j}\| \\
&\quad \le \|u_{iN+j} - S_j^{iN+j} u_{iN+j}\| + L_j\big[\|S_j^{iN+j-1} u_{iN+j} - S_j^{iN+j-1} u_{iN+j-1}\| \\
&\qquad + \|S_j^{iN+j-1} u_{iN+j-1} - u_{iN+j}\|\big] \\
&\quad \le \|u_{iN+j} - S_j^{iN+j} u_{iN+j}\| + L_j^2 \|u_{iN+j} - u_{iN+j-1}\| \\
&\qquad + L_j\big[\|S_j^{iN+j-1} u_{iN+j-1} - u_{iN+j-1}\| + \|u_{iN+j-1} - u_{iN+j}\|\big] \\
&\quad \to 0 \quad (\text{as } n \to \infty).
\end{aligned}$$

Similarly, we can prove that for each $i = 1, 2, \dots, N$,

$$\|Ax_{iN+j} - T_j^{iN+j} Ax_{iN+j}\| \to 0 \quad (i \to \infty). \tag{3.19}$$

Since T_j is uniformly $\tilde{L}_j$-Lipschitzian continuous, in the same way as above, we can also prove that

$$\|Ax_{iN+j} - T_j Ax_{iN+j}\| \to 0 \quad (\text{as } i \to \infty).$$

(V) Finally, we prove that $x_n \rightharpoonup x^*$, $u_n \rightharpoonup x^*$, and it is a solution of problem (*MSSFP*) (1.1).

In fact, since $\{u_n\}$ is bounded, there exists a subsequence $\{u_{n_i}\} \subset \{u_n\}$ such that $u_{n_i} \rightharpoonup x^* \in H_1$. Hence, for any positive integer $j = 1, 2, \dots, N$, there exists a subsequence $n_i(j) \subset n_i$ with $n_i(j) \bmod N = j$ such that $u_{n_i(j)} \rightharpoonup x^*$. Again from (3.17) we have that

$$\|u_{n_i(j)} - S_j u_{n_i(j)}\| \to 0, \qquad n_{i(j)} \to \infty. \tag{3.20}$$

Since S_j is demiclosed at zero, it follows that $x^* \in F(S_j)$. By the arbitrariness of $j = 1, 2, \dots, N$, we have

$$x^* \in C := \bigcap_{i=1}^{N} F(S_i).$$

Moreover, from (3.1) and (3.13) we have $x_{n_i} = u_{n_i} - \gamma A^*(T_{n_i(\bmod N)}^{n_i} - I)Ax_{n_i} \rightharpoonup x^*$. Since A is a linear bounded operator, it follows that $Ax_{n_i} \rightharpoonup Ax^*$. For any positive integer $k = 1, 2, \dots, N$, there exists a subsequence $x_{n_i(k)} \subset x_{n_i}$ with $n_i(k) (\bmod N) = k$ such that $Ax_{n_i(k)} \rightharpoonup Ax^*$ and $\|Ax_{n_i(k)} - T_k Ax_{n_i(k)}\| \to 0$. Since T_k is demiclosed at zero, we have $Ax^* \in F(T_k)$. By the arbitrariness of k, it follows that $Ax^* \in Q := \bigcap_{k=1}^{N} F(T_k)$. This together with $x^* \in C$ shows that $x^* \in \Gamma$, that is, x^* is a solution to the problem (*MSSFP*) (1.1).

Next we prove that $x_n \rightharpoonup x^*$ and $u_n \rightharpoonup x^*$.

In fact, assume that there exists another subsequence $u_{n_l} \subset u_n$ such that $u_{n_l} \rightharpoonup y^* \in \Gamma$ with $y^* \ne x^*$. Consequently, by virtue of the existence of $\lim_{n\to\infty} \|x_n - p\|$ and the Opial property of a Hilbert space, we have

$$\begin{aligned}\liminf_{n_i\to\infty}\|u_{n_i}-x^*\| &< \liminf_{n_i\to\infty}\|u_{n_i}-y^*\| \\ &= \liminf_{n\to\infty}\|u_n-y^*\| \liminf_{n_j\to\infty}\|u_{n_j}-y^*\| \\ &< \liminf_{n_j\to\infty}\|u_{n_j}-x^*\| = \liminf_{n\to\infty}\|u_n-x^*\| \\ &= \liminf_{n_i\to\infty}\|u_{n_i}-x^*\|.\end{aligned}$$

This is a contradiction. Therefore, $u_n \rightharpoonup x^*$. By (3.1) and (3.13), we have

$$x_n = u_n - \gamma A^*\big(T^n_{n(\mathrm{mod}\,N)} - I\big)Ax_n \rightharpoonup x^*.$$

This completes the proof of Theorem 3.1. □

Theorem 3.2 *Let H_1, H_2, A, $\{S_i\}$, $\{T_i\}$, C, Q be the same as in Theorem 3.1. For each $i = 1,2,\dots,N$, let T_i be a uniformly $\tilde{L}_i$-Lipschitzian and κ_i-asymptotically strictly pseudo-nonspreading mapping, S_i be a uniformly L_i-Lipschitzian and ϱ_i-asymptotically strictly pseudo-nonspreading mapping. Let $\{x_n\}$ be the sequence generated by*

$$\begin{cases} x_1 \in H_1 \text{ chosen arbitrarily}, \\ u_n = x_n + \gamma A^*(T^n_{n(\mathrm{mod}\,N)} - I)Ax_n, \\ x_{n+1} = (1-\alpha_n)u_n + \alpha_n S^n_{n(\mathrm{mod}\,N)}u_n, \end{cases}$$

where γ is a constant and $\gamma \in (0, \frac{1-\kappa}{\lambda})$, λ is the spectral of the operator A^*A, $\kappa = \max\{\kappa_1,\kappa_2,\dots,\kappa_N\}$ and $\{\alpha_n\}$ is a sequence in $(0,1-\varrho]$ with $\varrho = \max\{\varrho_1,\varrho_2,\dots,\varrho_N\}$. If $\Gamma \neq \emptyset$ and if there exists a positive integer j such that S_j is semicompact, then the sequence $\{x_n\}$ converges strongly to a point $x^* \in \Gamma$.

Proof Without loss of generality, we can assume that S_1 is semicompact. It follows from (3.17) that

$$\|u_{n_{i(1)}} - S_1 u_{n_{i(1)}}\| \to 0, \qquad n_{i(1)} \to \infty.$$

Therefore, there exists a subsequence of $\{u_{n_{i(1)}}\}$, which (for the sake of convenience) we still denote by $\{u_{n_{i(1)}}\}$, such that $u_{n_{i(1)}} \to u^* \in H_1$. Since $u_{n_{i(1)}} \rightharpoonup x^*$, $x^* = u^*$, and so $u_{n_{i(1)}} \to x^* \in \Gamma$. By virtue of $\lim_{n\to\infty}\|x_n - p\|$ exists, we know that

$$\lim_{n\to\infty}\|u_n - x^*\| = 0, \qquad \lim_{n\to\infty}\|x_n - x^*\| = 0,$$

that is, $\{u_n\}$ and $\{x_n\}$ both converge strongly to the point $x^* \in \Gamma$. This completes the proof of Theorem 3.2. □

4 Applications

In this section we shall utilize the results presented in Section 3 to study the *hierarchical variational inequality problem*.

Let H be a real Hilbert space, S_i, $i = 1,2,\ldots,N$, be uniformly L_i-Lipschitzian and ϱ_i-asymptotically strictly pseudo-nonspreading mappings with $\mathcal{F} := \bigcap_{i=1}^{\infty} F(S_i) \neq \emptyset$. Let $T : H \to H$ be a nonspreading mapping. The so-called *hierarchical variational inequality problem for a finite family of mappings* $\{S_i\}$ *with respect to the mapping* T is to find an $x^* \in \mathcal{F}$ such that

$$\langle x^* - Tx^*, x^* - x\rangle \leq 0, \quad \forall x \in \mathcal{F}. \tag{4.1}$$

It is easy to see that (4.1) is equivalent to the following fixed point problem:

$$\text{find } x^* \in \mathcal{F} \text{ such that } x^* = P_{\mathcal{F}} Tx^*, \tag{4.2}$$

where $P_{\mathcal{F}}$ is the metric projection from H onto $\mathcal{F}$. Letting $C = \mathcal{F}$ and $Q = F(P_{\mathcal{F}} T)$ (the fixed point set of $P_{\mathcal{F}} T$) and $A = I$ (the identity mapping on H), problem (4.2) is equivalent to the following *multi-set split feasibility problem*:

$$\text{find } x^* \in C \text{ such that } x^* \in Q. \tag{4.3}$$

Hence from Theorem 3.1 we have the following theorem.

Theorem 4.1 *Let H, $\{S_i\}$, T, C, Q be the same as above. Let $\{x_n\}$, $\{u_n\}$ be the sequences defined by*

$$\begin{cases} x_1 \in H_1 \text{ chosen arbitrarily}, \\ u_n = x_n + \gamma(T - I)x_n, \quad n \geq 1, \\ x_{n+1} = (1 - \alpha_n)u_n + \alpha_n S^n_{n (\operatorname{mod} N)} u_n, \end{cases} \tag{4.4}$$

where γ is a constant and $\gamma \in (0,1)$, and $\{\alpha_n\}$ is a sequence in $(0,1-\varrho]$ with $\varrho = \max\{\varrho_1,\varrho_2,\ldots,\varrho_N\}$. If $\Gamma \neq \emptyset$, then $\{x_n\}$ converges weakly to a solution of hierarchical variational inequality problem (4.1).

Proof In fact, by the assumption that T is a nonspreading mapping, T is κ-strictly pseudo-nonspreading with $\kappa = 0$. Taking $N = 1$ and $A = I$ in Theorem 3.1, by the same method as that given in Theorem 3.1, we can prove that $\{x_n\}$ converges weakly to a point $x^* \in \Gamma$, which is a solution of hierarchical variational inequality problem (4.1) immediately. □

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally and significantly to this research work. All authors read and approved the final manuscript.

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References

1. Censor, Y, Elfving, T: A multiprojection algorithm using Bregman projections in a product space. Numer. Algorithms **8**(2-4), 221-239 (1994)
2. Byrne, C: Iterative oblique projection onto convex sets and the split feasibility problem. Inverse Probl. **18**(2), 441-453 (2002)
3. Censor, Y, Bortfeld, T, Martin, B, Trofimov, A: A unified approach for inversion problems in intensity-modulated radiation therapy. Phys. Med. Biol. **51**(10), 2353-2365 (2006)
4. Censor, Y, Elfving, T, Kopf, N, Bortfeld, T: The multiple-sets split feasibility problem and its applications for inverse problem and its applications. Inverse Probl. **21**(6), 2071-2084 (2005)
5. Censor, Y, Motova, A, Segal, A: Perturbed projections and subgradient projections for the multiple-sets split feasibility problem. J. Math. Anal. Appl. **327**(2), 1244-1256 (2007)
6. Xu, HK: A variable Krasnosel'skii-Mann algorithm and the multiple-set split feasibility problem. Inverse Probl. **22**(6), 2021-2034 (2006)
7. Chang, S-S, Cho, YJ, Kim, JK, Zhang, WB, Yang, L: Multiple-set split feasibility problems for asymptotically strict pseudocontractions. Abstr. Appl. Anal. **2012**, Article ID 491760 (2012). doi:10.1155/2012/491760
8. Kohsaka, F, Takahashi, W: Fixed point theorems for a class of nonlinear mappings relate to maximal monotone operators in Banach spaces. Arch. Math. **91**, 166-177 (2008)
9. Kohsaka, F, Takahashi, W: Existence and approximation of fixed points of firmly nonexpansive-type mappings in Banach spaces. SIAM J. Optim. **19**, 824-835 (2008)
10. Iemoto, S, Takahashi, W: Approximating common fixed points of nonexpansive mappings and nonspreading mappings in a Hilbert space. Nonlinear Anal. **71**, 2080-2089 (2009)
11. Kurokawa, Y, Takahashi, W: Weak and strong convergence theorems for nonspreading mappings in Hilbert spaces. Nonlinear Anal. **73**, 1562-1568 (2010)
12. Browder, FE, Petryshyn, WV: Construction of fixed points of nonlinear mappings in Hilbert spaces. J. Math. Anal. Appl. **20**, 197-228 (1967)
13. Liu, QH: Convergence theorems of the sequence of iterates for asymptotically demicontractive and hemicontractive mappings. Nonlinear Anal., Theory Methods Appl. **26**(11), 1835-1842 (1996)
14. Kim, TH, Xu, HK: Convergence of the modified Mann's iteration method for asymptotically strict pseudo-contractions. Nonlinear Anal., Theory Methods Appl. **68**, 2828-2836 (2008)
15. Inchan, I, Nammanee, K: Strong convergence theorems by hybrid method for asymptotically image-strict pseudocontractive mapping in Hilbert space. Nonlinear Anal. **3**, 380-385 (2009)
16. Zhou, H: Demiclosedness principle with applications for asymptotically pseudo-contractions in Hilbert spaces. Nonlinear Anal. **70**, 3140-3145 (2009)
17. Cho, SY: Iterative processes for common fixed points of two different families of mappings with applications. J. Glob. Optim. **57**, 1429-1446 (2013)
18. Ge, C: A hybrid algorithm with variable coefficients for asymptotically pseudocontractive mappings in the intermediate sense on unbounded domains. Nonlinear Anal. **75**, 2859-2866 (2012)
19. Osilike, MO, Isiogugu, FO: Weak and strong convergence theorems for nonspreading-type mappings in Hilbert spaces. Nonlinear Anal. **74**, 1814-1822 (2011)
20. Goebel, K, Kirk, WA: A fixed point theorem for asymptotically nonexpansive mappings. Proc. Am. Math. Soc. **35**, 171-174 (1972)

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