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Exponential convergence of Cohen-Grossberg neural networks with continuously distributed leakage delays

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Abstract

This paper is concerned with the global exponential convergence of Cohen-Grossberg neural networks with continuously distributed leakage delays. By using the Lyapunov functional method and differential inequality techniques, we propose a new approach to establishing some sufficient conditions ensuring that all solutions of the networks converge exponentially to the zero point. Our results complement some recent ones.

MSC: 34C25; 34K13; 34K25

Keywords: Cohen-Grossberg neural network; global exponential convergence; continuously distributed delay; leakage term

1 Introduction

It is well known that Cohen-Grossberg neural networks (CGNNs) have been successfully applied in many fields such as pattern recognition, parallel computing, associative memory, and combinatorial optimization (see [1–5]). Such applications heavily depend on the global exponential convergence behaviors, because the exponential convergent rate can be unveiled. Many good results on the problem of the global exponential convergence of the equilibriums and periodic solutions of for CGNNs are given in the literature. We refer the reader to [6–13] and the references cited therein. Recently, in real applications, a typical time delay called leakage (or 'forgetting') delay has been introduced in the negative feedback terms of the neural network system, and these terms are variously known as forgetting or leakage terms (see [14–16]). Subsequently, Gopalsamy [17] investigated the stability on the equilibrium for the bidirectional associative memory (BAM) neural networks with constant delay in the leakage term. Following this, the authors of [18–22] dealt with the existence and stability of equilibrium and periodic solutions for neuron networks model involving constant leakage delays. In particular, Peng [23] established some delay dependent criteria for the existence and global attractive periodic solutions of the bidirectional associative memory neural network with continuously distributed delays in the leakage terms. However, to the best of our knowledge, few authors have considered the exponential convergence behavior for all solutions of CGNNs with continuously distributed delays in the leakage terms. Motivated by the arguments above, in the present paper, we shall consider the following CGNNs with time-varying coefficients and continuously distributed delays in the leakage terms:

$$x_i'(t) = -a_i\big(t, x_i(t)\big)\Bigg[b_i\bigg(t, \int_0^\infty \delta_i(s)x_i(t-s)\,ds\bigg) - \sum_{j=1}^n c_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big) \\ - \sum_{j=1}^n d_{ij}(t)\int_0^\infty K_{ij}(u)g_j\big(x_j(t-u)\big)\,du + I_i(t)\Bigg], \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where a_i and b_i are continuous functions on R^2, δ_i, τ_{ij}, f_j, g_j, c_{ij}, d_{ij} and I_i are continuous functions on R; n corresponds to the number of units in a neural network; $x_i(t)$ denotes the potential (or voltage) of cell i at time t; a_i represents an amplification function; b_i is an appropriately behaved function; $c_{ij}(t)$ and $d_{ij}(t)$ denote the strengths of connectivity between cell i and j at time t, respectively; the activation functions $f_i(\cdot)$ and $g_i(\cdot)$ show how the ith neuron reacts to the input, $\tau_{ij}(t) \geq 0$ corresponds to the transmission delays, $K_{ij}(u)$ and $\delta_i(u) \geq 0$ correspond to the transmission delay kernels, and $I_i(t)$ denotes the ith component of an external input source introduced from outside the network to cell i at time t for $i, j \in F = \{1, 2, \ldots, n\}$.

Throughout this paper, for $i, j \in F$, it will be assumed that $h_i : [0, +\infty) \to [0, +\infty)$ and $K_{ij} : [0, +\infty) \to R$ are continuous functions, and there exist constants τ_{ij}^+, $\overline{I_i}$, $\overline{c_{ij}}$, and $\overline{d_{ij}}$ such that

$$\tau_{ij}^+ = \sup_{t\in R} \tau_{ij}(t), \qquad \overline{I_i} = \sup_{t\in R}\big|I_i(t)\big|, \qquad \overline{c_{ij}} = \sup_{t\in R}\big|c_{ij}(t)\big|, \qquad \overline{d_{ij}} = \sup_{t\in R}\big|d_{ij}(t)\big|. \tag{1.2}$$

We also make the following assumptions.

(H_1) For each $j \in F$, there exist nonnegative constants β, α, $\tilde{L}_j$ and L_j such that

$$0 \leq \beta \leq 1, 0 \leq \alpha \leq 1, \quad \big|f_j(u)\big| \leq \tilde{L}_j|u|^\beta, \qquad \big|g_j(u)\big| \leq L_j|u|^\alpha \\ \text{for all } u \in R. \tag{1.3}$$

(H_2) For $i \in F$, there exist positive constants $\underline{a_i}$ and $\overline{a_i}$ such that

$$\underline{a_i} \leq a_i(t, u) \leq \overline{a_i} \quad \text{for all } t > 0, u \in R.$$

(H_3) For $i \in F$, $b_i(t, 0) \equiv 0$, and there exist positive constants $\underline{b_i}$ and $\overline{b_i}$ such that

$$\underline{b_i}|u - v| \leq \operatorname{sgn}(u - v)\big(b_i(t, u) - b_i(t, v)\big) \leq \overline{b_i}|u - v| \quad \text{for all } t > 0, u, v \in R.$$

(H_4) For all $t > 0$ and $i, j \in F$, there exist constants $\eta > 0$ and $\lambda > 0$ such that

$$\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds < +\infty, \qquad \int_0^\infty \big|K_{ij}(u)\big|e^{\lambda u}\,du < +\infty$$

and

$$-\eta > -\bigg[\underline{a_i}\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds - \lambda\bigg(1 + \overline{a_i}\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg) \\ - \overline{a_i}\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{a_i}\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds\bigg]$$

$$+\overline{a_i}\Bigg[\sum_{j=1}^{n}\tilde{L}_j\bigg(\big|c_{ij}(t)\big|e^{\lambda\beta\tau_{ij}(t)}+\overline{a_i}\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{c_{ij}}e^{\lambda\beta\tau_{ij}^+}\bigg)e^{\lambda(1-\beta)t}$$
$$+\sum_{j=1}^{n}L_j\int_0^\infty\big|K_{ij}(u)\big|e^{\lambda\alpha u}\,du\bigg(\big|d_{ij}(t)\big|+\overline{d_{ij}}\overline{a_i}\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg)e^{\lambda(1-\alpha)t}\Bigg].$$

(H_5) $I_i(t)=O(e^{-\lambda t})$ $(t\to\pm\infty)$, $i\in F$.

The initial conditions associated with system (1.1) are of the form

$$x_i(s)=\varphi_i(s),\quad s\in(-\infty,0],i\in F,\tag{1.4}$$

where $\varphi_i(\cdot)$ denotes a real-valued bounded continuous function defined on $(-\infty,0]$.

The remaining part of this paper is organized as follows. In Section 2, we present some new sufficient conditions to ensure that all solutions of CGNNs (1.1) with initial conditions (1.4) converge exponentially to the zero point. In Section 3, we shall give some examples and remarks to illustrate our results obtained in the previous sections.

2 Main results

Theorem 2.1 *Let* (H_1)-(H_5) *hold. Then, for every solution* $Z(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$ *of CGNNs* (1.1) *with initial conditions* (1.4), *there exists a positive constant* K *such that*

$$\big|x_i(t)\big|\le Ke^{-\lambda t}\quad\text{for all } t>0,i\in F.$$

Proof Let $Z(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$ be a solution of system (1.1) with initial conditions (1.4), and let

$$X_i(t)=e^{\lambda t}x_i(t),\quad i\in F.$$

In view of (1.1), we have

$$\begin{aligned}X_i'(t)={}&\lambda X_i(t)+e^{\lambda t}a_i\big(t,x_i(t)\big)\Bigg[-b_i\bigg(t,\int_0^\infty\delta_i(s)e^{\lambda(s-t)}X_i(t-s)\,ds\bigg)\\&+\sum_{j=1}^{n}c_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big)+\sum_{j=1}^{n}d_{ij}(t)\int_0^\infty K_{ij}(u)g_j\big(x_j(t-u)\big)\,du-I_i(t)\Bigg]\\={}&\lambda X_i(t)+e^{\lambda t}a_i\big(t,x_i(t)\big)\Bigg[-b_i\bigg(t,\int_0^\infty\delta_i(s)e^{\lambda(s-t)}X_i(t)\,ds\bigg)\\&+\bigg(b_i\bigg(t,\int_0^\infty\delta_i(s)e^{\lambda(s-t)}X_i(t)\,ds\bigg)-b_i\bigg(t,\int_0^\infty\delta_i(s)e^{\lambda(s-t)}X_i(t-s)\,ds\bigg)\bigg)\\&+\sum_{j=1}^{n}c_{ij}(t)f_j\big(x_j\big(t-\tau_{ij}(t)\big)\big)\\&+\sum_{j=1}^{n}d_{ij}(t)\int_0^\infty K_{ij}(u)g_j\big(x_j(t-u)\big)\,du-I_i(t)\Bigg],\quad i=1,2,\dots,n.\end{aligned}\tag{2.1}$$

Let

$$M = \max_{i=1,2,\dots,n} \sup_{s\le 0}\left\{e^{\lambda s}\left|\varphi_i(s)\right|\right\}. \tag{2.2}$$

From (1.2) and (H_5), we can choose a positive constant $K > M + 1$ such that

$$\eta > \frac{[1 + \overline{a_i}\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds]\overline{a_i}\sup_{t\in R}|e^{\lambda t}I_i(t)|}{K} \quad \text{for all } t > 0, i \in F. \tag{2.3}$$

Then it is easy to see that

$$\left|X_i(t)\right| \le M < K \quad \text{for all } t \le 0, i = 1, 2, \dots, n.$$

We now claim that

$$\left|X_i(t)\right| < K \quad \text{for all } t > 0, i \in F. \tag{2.4}$$

Otherwise, one of the following two cases must occur.

(1) There exist $i \in F$ and $t^* > 0$ such that

$$X_i\left(t^*\right) = K, \qquad \left|X_j(t)\right| < K \quad \text{for all } t < t^*, j \in F. \tag{2.5}$$

(2) There exist $i \in F$ and $t^{**} > 0$ such that

$$X_i\left(t^{**}\right) = -K, \qquad \left|X_j(t)\right| < K \quad \text{for all } t < t^{**}, j \in F. \tag{2.6}$$

Now, we distinguish two cases to finish the proof.

Case (1). If (2.5) holds. Then, from (2.1), (2.3), and (H_1)-(H_4), we have

$$\begin{aligned}
0 &\le X_i'\left(t^*\right) \\
&= \lambda X_i\left(t^*\right) + e^{\lambda t^*} a_i\left(t^*, x_i\left(t^*\right)\right)\Bigg[-b_i\left(t^*, \int_0^\infty \delta_i(s)e^{\lambda(s-t^*)}X_i\left(t^*\right)ds\right) \\
&\quad + \left(b_i\left(t^*, \int_0^\infty \delta_i(s)e^{\lambda(s-t^*)}X_i\left(t^*\right)ds\right) - b_i\left(t^*, \int_0^\infty \delta_i(s)e^{\lambda(s-t^*)}X_i\left(t^*-s\right)ds\right)\right) \\
&\quad + \sum_{j=1}^n c_{ij}\left(t^*\right)f_j\left(x_j\left(t^* - \tau_{ij}\left(t^*\right)\right)\right) \\
&\quad + \sum_{j=1}^n d_{ij}\left(t^*\right)\int_0^\infty K_{ij}(u)g_j\left(x_j\left(t^*-u\right)\right)du - I_i\left(t^*\right)\Bigg] \\
&\le \lambda X_i\left(t^*\right) - \underline{a_i}\,\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds X_i\left(t^*\right) + \overline{a_i}\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\int_{t^*-s}^{t^*} X_i'(u)\,du\,ds \\
&\quad + \overline{a_i}\sum_{j=1}^n \left|c_{ij}\left(t^*\right)\right|\tilde{L}_j e^{\lambda\beta\tau_{ij}(t^*)}e^{\lambda(1-\beta)t^*}\left|X_j\left(t^* - \tau_{ij}\left(t^*\right)\right)\right|^\beta \\
&\quad + \overline{a_i}\sum_{j=1}^n \left|d_{ij}\left(t^*\right)\right|L_j e^{\lambda(1-\alpha)t^*}\int_0^\infty \left|K_{ij}(u)\right|e^{\lambda\alpha u}\left|X_j\left(t^*-u\right)\right|^\alpha du + \overline{a_i}e^{\lambda t^*}\left|I_i\left(t^*\right)\right|
\end{aligned}$$

$$
\begin{aligned}
&\le \lambda X_i(t^*) - \underline{a_i}\,\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds X_i(t^*) + \overline{a_i}\,\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\int_{t^*-s}^{t^*}\bigg|\lambda X_i(u) \\
&\quad + e^{\lambda u}a_i\big(u,x_i(u)\big)\bigg[-b_i\bigg(u,\int_0^\infty \delta_i(v)e^{\lambda(v-u)}X_i(u-v)\,dv\bigg) + \sum_{j=1}^n c_{ij}(u)f_j\big(x_j\big(u-\tau_{ij}(u)\big)\big) \\
&\quad + \sum_{j=1}^n d_{ij}(u)\int_0^\infty K_{ij}(v)g_j\big(x_j(u-v)\big)\,dv - I_i(u)\bigg]\bigg|\,du\,ds \\
&\quad + \overline{a_i}\sum_{j=1}^n \big|c_{ij}(t^*)\big|\tilde{L}_j e^{\lambda\beta\tau_{ij}(t^*)}e^{\lambda(1-\beta)t^*}\big|X_j\big(t^*-\tau_{ij}(t^*)\big)\big|^\beta \\
&\quad + \overline{a_i}\sum_{j=1}^n \big|d_{ij}(t^*)\big|L_j e^{\lambda(1-\alpha)t^*}\int_0^\infty \big|K_{ij}(u)\big|e^{\lambda\alpha u}\big|X_j(t^*-u)\big|^\alpha\,du + \overline{a_i}e^{\lambda t^*}\big|I_i(t^*)\big| \\
&\le \lambda X_i(t^*) - \underline{a_i}\,\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds X_i(t^*) + \lambda\overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds X_i(t^*) \\
&\quad + \overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{a_i}\,\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds X_i(t^*) \\
&\quad + \overline{a_i}\,\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\int_{t^*-s}^{t^*}\overline{a_i}\bigg[\sum_{j=1}^n \overline{c_{ij}}\tilde{L}_j e^{\lambda\beta\tau_{ij}^+}e^{\lambda(1-\beta)u}\big|X_j\big(u-\tau_{ij}(u)\big)\big|^\beta \\
&\quad + \sum_{j=1}^n \overline{d_{ij}}L_j e^{\lambda(1-\alpha)u}\int_0^\infty \big|K_{ij}(v)\big|e^{\lambda\alpha v}\big|X_j(u-v)\big|^\alpha\,dv + \sup_{t\in R}\big|e^{\lambda t}I_i(t)\big|\bigg]\,du\,ds \\
&\quad + \overline{a_i}\sum_{j=1}^n \big|c_{ij}(t^*)\big|\tilde{L}_j e^{\lambda\beta\tau_{ij}(t^*)}e^{\lambda(1-\beta)t^*}\big|X_j\big(t^*-\tau_{ij}(t^*)\big)\big|^\beta \\
&\quad + \overline{a_i}\sum_{j=1}^n \big|d_{ij}(t^*)\big|L_j e^{\lambda(1-\alpha)t^*}\int_0^\infty \big|K_{ij}(u)\big|e^{\lambda\alpha u}\big|X_j(t^*-u)\big|^\alpha\,du + \overline{a_i}e^{\lambda t^*}\big|I_i(t^*)\big| \\
&\le -\bigg[\underline{a_i}\,\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds - \lambda\bigg(1+\overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg) \\
&\quad - \overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{a_i}\,\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds\bigg]X_i(t^*) \\
&\quad + \overline{a_i}\bigg[\sum_{j=1}^n \tilde{L}_j\bigg(\big|c_{ij}(t^*)\big|e^{\lambda\beta\tau_{ij}(t^*)} + \overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{c_{ij}}e^{\lambda\beta\tau_{ij}^+}\bigg)e^{\lambda(1-\beta)t^*} \\
&\quad + \sum_{j=1}^n L_j\int_0^\infty \big|K_{ij}(u)\big|e^{\lambda\alpha u}\,du\bigg(\big|d_{ij}(t^*)\big| + \overline{d_{ij}}\,\overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg)e^{\lambda(1-\alpha)t^*}\bigg]K \\
&\quad + \bigg[1+\overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg]\overline{a_i}\sup_{t\in R}\big|e^{\lambda t}I_i(t)\big| \\
&= \bigg\{-\bigg[\underline{a_i}\,\underline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds - \lambda\bigg(1+\overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\bigg) \\
&\quad - \overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{a_i}\,\overline{b_i}\int_0^\infty \delta_i(s)e^{\lambda s}\,ds\bigg] \\
&\quad + \overline{a_i}\bigg[\sum_{j=1}^n \tilde{L}_j\bigg(\big|c_{ij}(t^*)\big|e^{\lambda\beta\tau_{ij}(t^*)} + \overline{a_i}\,\overline{b_i}\int_0^\infty s\delta_i(s)e^{\lambda s}\,ds\overline{c_{ij}}e^{\lambda\beta\tau_{ij}^+}\bigg)e^{\lambda(1-\beta)t^*}
\end{aligned}
$$

$$\begin{aligned}
&+\sum_{j=1}^{n}L_j\int_0^{\infty}\bigl|K_{ij}(u)\bigr|e^{\lambda\alpha u}\,du\biggl(\bigl|d_{ij}(t^*)\bigr|+\overline{d_{ij}}\overline{a_i}\overline{b_i}\int_0^{\infty}s\delta_i(s)e^{\lambda s}\,ds\biggr)e^{\lambda(1-\alpha)t^*}\Biggr]\Biggr\}K\\
&+\biggl[1+\overline{a_i}\overline{b_i}\int_0^{\infty}s\delta_i(s)e^{\lambda s}\,ds\biggr]\overline{a_i}\sup_{t\in R}\bigl|e^{\lambda t}I_i(t)\bigr|\\
<{}&-\eta K+\biggl[1+\overline{a_i}\overline{b_i}\int_0^{\infty}s\delta_i(s)e^{\lambda s}\,ds\biggr]\overline{a_i}\sup_{t\in R}\bigl|e^{\lambda t}I_i(t)\bigr|\\
<{}&0.
\end{aligned}$$

This contradiction implies that (2.5) does not hold.

Case (2). If (2.6) holds, then, from (2.1), (2.3), and (H_1)-(H_4), by using a similar argument as in Case (1), we can derive a contradiction, which shows that (2.6) does not hold.

Therefore, (2.4) is proved and

$$\bigl|x_i(t)\bigr|\le Ke^{-\lambda t}\quad\text{for all } t>0, i\in F.$$

This implies that the proof of Theorem 2.1 is now completed. □

3 An example

Example 3.1 Consider the following CGNNs with time-varying delays in the leakage terms:

$$\begin{cases}
x_1'(t)=-(2+e^{\cos^2 t}\frac{1}{10\pi}\arctan x_1(t))[(4-\frac{|t||\sin t|}{1+2|t|})\int_0^{\infty}\delta_1(s)x_1(t-s)\,ds\\
\quad+\frac{1}{70}\frac{|t|\sin t}{1+40|t|}f_1(x_1(t-2\sin^2 t))+\frac{1}{70}\frac{|t|\sin t}{1+36|t|}\\
\quad\cdot f_2(x_2(t-3\sin^2 t))+\frac{1}{70}\frac{|t|\sin t}{1+40|t|}\int_0^{\infty}e^{-u}g_1(x_j(t-u))\,du\\
\quad+\frac{1}{70}\frac{|t|^2\sin t}{1+36|t|^2}\int_0^{\infty}e^{-u}g_2(x_j(t-u))\,du+20{,}000e^{-3t}\sin t],\\
x_2'(t)=-(2+e^{\sin^2 t}\frac{1}{10\pi}\arctan x_2(t))[(4-\frac{|t||\cos t|}{1+2|t|})\int_0^{\infty}\delta_2(s)x_2(t-s)\,ds\\
\quad+\frac{1}{70}\frac{|t|\cos t}{1+40|t|}f_1(x_1(t-2\sin^2 t))\\
\quad+\frac{1}{70}\frac{|t|\cos t}{1+36|t|}f_2(x_2(t-5\sin^2 t))+\frac{1}{70}\frac{|t|\cos t}{1+40|t|}\int_0^{\infty}e^{-u}g_1(x_j(t-u))\,du\\
\quad+\frac{1}{70}\frac{|t|\cos t}{1+36|t|}\int_0^{\infty}e^{-u}g_2(x_j(t-u))\,du+30{,}000e^{-t}\cos t],
\end{cases}\tag{3.1}$$

where $f_i(x)=g_i(x)=x\sin^{2i}x$, $\delta_i(t)=e^{-10t}$, $i=1,2$.

It follows that

$$1\le\underline{a_i}\le\overline{a_i}\le 3,\qquad 3\le\underline{b_i}\le\overline{b_i}\le 4,\quad i=1,2$$

and

$$\underline{b_i}|u|\le\operatorname{sgn}(u)b_i(t,u)\quad\text{for all } t,u\in R, i=1,2.$$

Define a continuous function $\Gamma_i(\omega)$ by setting

$$\begin{aligned}
\Gamma_i(\omega)={}&-\biggl[\underline{a_i}\underline{b_i}\int_0^{\infty}\delta_i(s)e^{\omega s}\,ds-\omega\biggl(1+\overline{a_i}\overline{b_i}\int_0^{\infty}s\delta_i(s)e^{\omega s}\,ds\biggr)\\
&-\overline{a_i}\overline{b_i}\int_0^{\infty}s\delta_i(s)e^{\omega s}\,ds\overline{a_i}\overline{b_i}\int_0^{\infty}\delta_i(s)e^{\omega s}\,ds\biggr]+\overline{a_i}\Biggl[\sum_{j=1}^{n}\tilde{L}_j\biggl(\bigl|c_{ij}(t)\bigr|e^{\omega\beta\tau_{ij}(t)}
\end{aligned}$$

$$+ \overline{a_i}\overline{b_i} \int_0^\infty s\delta_i(s)e^{\omega s}\,ds\overline{c_{ij}}e^{\omega\beta\tau_{ij}^+}\bigg)e^{\omega(1-\beta)t} + \sum_{j=1}^n L_j \int_0^\infty |K_{ij}(u)|e^{\omega\alpha u}\,du\bigg(|d_{ij}(t)|$$

$$+ \overline{d_{ij}}\overline{a_i}\overline{b_i} \int_0^\infty s\delta_i(s)e^{\omega s}\,ds\bigg)e^{\omega(1-\alpha)t}\bigg] \quad \text{for all } t > 0, i = 1, 2.$$

According to the continuity of $\Gamma_i(\omega)$ and $\Gamma_i(0) < 0$, we can choose constants $\eta = 0.1$ and $\lambda > 0$ such that

$$\Gamma_i(\lambda) < -\eta \quad \text{for all } t > 0, i = 1, 2,$$

which implies that the CGNNs (3.1) satisfied (H_1)-(H_5). Hence, from Theorem 2.1, all solutions of the CGNNs (3.1) with initial value $(\varphi_1(x), \varphi_2(x))$ converge exponentially to the zero point $(0, 0)$.

Remark 3.1 It is easy to check that the results in [17–23] and [24–34] are invalid for the global exponential convergence of (3.1), since the leakage delays are continuously distributed.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

ZC gave the proof of Theorem 2.1 and drafted the manuscript. SG proved and gave the example to illustrate the effectiveness of the obtained results. All authors read and approved the final manuscript.

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