Journal of Inequalities and Applications
a SpringerOpen Journal

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Remarks on contractive mappings via Ω-distance

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Abstract
Very recently, some authors discovered that some fixed point results in the context of a G-metric space can be derived from the fixed point results in the context of a quasi-metric space and hence the usual metric space. In this article, we investigate some fixed point results in the framework of a G-metric space via Ω-distance that cannot be obtained by the usual fixed point results in the literature. We also add an application to illustrate our results.
MSC: 47H10; 54H25; 46J10; 46J15

Keywords: Ω-distance; fixed point; G-metric space

1 Introduction and preliminaries

Very recently, Jleli and Samet [1] and Samet *et al.* [2] proved that some fixed point results in the setting of G-metric spaces, introduced by Sims and Mustafa [3], are consequences of the well-known fixed point theorem in the context of the usual metric space. Indeed, authors in [1, 2] noticed that $G(x,y,y) = q(x,y)$ is a quasi-metric and obtained that the results are just a characterization of existence results in the framework of a quasi-metric. On the other hand, a G-metric was introduced as a generalization of the (usual) metric. Basically, G-metrics claim the geometry of three points instead of two points. Consequently, Jleli and Samet [1] and Samet *et al.* [2] concluded that if the expression in the fixed point theorem can be reduced to two points, then it can be written as a consequence of the related existence result in the literature.

Recently, Saadati *et al.* [4] introduced the concept of Ω-distance on a complete G-metric space as a generalized notion of ω-distance due to Kada *et al.* [5]. In these papers, the authors investigate the existence/uniqueness of a fixed point of certain operators in this setting. In this paper, we revise some published papers (see, *e.g.*, [6, 7]) and improve the statements in a way that cannot be manipulated by the techniques used in [1, 2] (see also [8–10]).

We first recall some necessary definitions and basic results on the topics in the literature.

Definition 1 ([3]) Let X be a non-empty set. A function $G : X \times X \times X \to [0,\infty)$ is called a G-metric if the following conditions are satisfied:

(i) $G(x,y,z) = 0$ if $x = y = z$ (coincidence),
(ii) $G(x,x,y) > 0$ for all $x,y \in X$, where $x \neq y$,
(iii) $G(x,x,z) \leq G(x,y,z)$ for all $x,y,z \in X$, with $z \neq y$,
(iv) $G(x,y,z) = G(p\{x,y,z\})$, where p is a permutation of x, y, z (symmetry),

(v) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).

A G-metric is said to be symmetric if $G(x,y,y) = G(y,x,x)$ for all $x,y \in X$.

Definition 2 ([3]) Suppose that (X,G) is a G-metric space.

(1) A sequence $\{x_n\}$ in X is said to be G-Cauchy sequence if, for each $\varepsilon > 0$, there exists a positive integer n_0 such that for all $n,m,l \geq n_0$, $G(x_n,x_m,x_l) < \varepsilon$.
(2) A sequence $\{x_n\}$ in X is said to be G-convergent to a point $x \in X$ if, for each $\varepsilon > 0$, there exists a positive integer n_0 such that for all $m,n \geq n_0$, $G(x_m,x_n,x) < \varepsilon$.

Definition 3 ([4]) Let (X,G) be a G-metric space. Then a function $\Omega : X \times X \times X \longrightarrow [0,\infty)$ is called an Ω-distance on X if the following conditions are satisfied:

(a) $\Omega(x,y,z) \leq \Omega(x,a,a) + \Omega(a,y,z)$ for all $x,y,z,a \in X$,
(b) $\Omega(x,y,\cdot), \Omega(x,\cdot,y) : X \to [0,\infty)$ are lower semi-continuous for any $x,y \in X$,
(c) for each $\varepsilon > 0$, there exists $\delta > 0$ such that $\Omega(x,a,a) \leq \delta$ and $\Omega(a,y,z) \leq \delta$ imply $G(x,y,z) \leq \varepsilon$.

Example 4 ([4]) Suppose that (X,d) is a metric space. Let $G : X^3 \longrightarrow [0,\infty)$ be defined as follows:

$$G(x,y,z) = \max\{d(x,y), d(y,z), d(x,z)\}$$

for all $x,y,z \in X$. Then one can easily show that $\Omega = G$ is an Ω-distance on X.

Example 5 ([4]) Let $X = \mathbb{R}$ and (X,G) be a G-metric, where

$$G(x,y,z) = \frac{1}{3}\big(|x-y| + |y-z| + |x-z|\big)$$

for all $x,y,z \in X$. If we define $\Omega : \mathbb{R}^3 \longrightarrow [0,\infty)$ as follows:

$$\Omega(x,y,z) = \frac{1}{3}\big(|z-x| + |x-y|\big)$$

for all $x,y,z \in X$, then it is an Ω-distance on $\mathbb{R}$.

We refer, *e.g.*, to [4, 11] for more details and examples on the topic.

Lemma 6 [4] *Suppose that (X,G) is a G-metric space and Ω is an Ω-distance on X. Let $\{x_n\}$, $\{y_n\}$ be sequences in X and $\{\alpha_n\}$, $\{\beta_n\}$ be sequences in $[0,\infty)$ converging to zero and $x,y,z,a \in X$. Then*

(a) *if $\Omega(y,x_n,x_n) \leq \alpha_n$ and $\Omega(x_n,y,z) \leq \beta_n$ for $n \in \mathbb{N}$, then $G(y,y,z) < \varepsilon$, and hence $y = z$;*
(b) *if $\Omega(y_n,x_n,x_n) \leq \alpha_n$ and $\Omega(x_n,y_m,z) \leq \beta_n$ for $m > n$, then $G(y_n,y_m,z) \to 0$, and hence $y_n \to z$;*
(c) *if $\Omega(x_n,x_m,x_l) \leq \alpha_n$ for any $l,m,n \in \mathbb{N}$ with $n \leq m \leq l$, then $\{x_n\}$ is a G-Cauchy sequence;*
(d) *if $\Omega(x_n,a,a) \leq \alpha_n$ for any $n \in \mathbb{N}$, then $\{x_n\}$ is a G-Cauchy sequence.*

Definition 7 ([4]) Suppose that (X,G) is a G-metric space and Ω is an Ω-distance on X. (X,G) is called Ω-bounded if there is a constant $C > 0$ with $\Omega(x,y,z) \leq C$ for all $x,y,z \in X$.

Definition 8 Let $(X, \leq)$ be a partially ordered set. A self-mapping $T : X \to X$ is said to be non-decreasing if, for $x, y \in X$,

$$x \leq y \quad \Longrightarrow \quad T(x) \leq T(y).$$

The tripled $(X, G, \leq)$ is called a partially ordered G-metric space if $(X, \leq)$ is a partially ordered set endowed with a G-metric on X; see also [12, 13].

2 Fixed point theorems on partially ordered *G*-metric spaces

We start this section with the following classes of mappings:

$$\Phi = \{\phi \mid \phi : [0,\infty) \to [0,\infty) \text{ continuous, non-decreasing}\} \quad \text{and}$$
$$\Psi = \{\psi \mid \psi : [0,\infty) \to [0,\infty) \text{ continuous, non-decreasing}\}$$

with $\phi^{-1}(\{0\}) = \psi^{-1}(\{0\}) = \{0\}$.

Definition 9 Let $(X, \leq)$ be a partially ordered space. Suppose that there exists a G-metric on X such that (X, G) is a complete G-metric space. A self-mapping $T : X \to X$ is said to be a generalized weak-contraction mapping if it satisfies the following condition:

$$\psi\bigl(\Omega\bigl(Tx, T^2x, Ty\bigr)\bigr) \leq \psi\bigl(\Omega(x, Tx, y)\bigr) - \phi\bigl(\Omega(x, Tx, y)\bigr) \quad \text{for all } x, y \in X, \text{ with } x \leq y,$$

where $\psi \in \Psi$ and $\phi \in \Phi$.

Theorem 10 *Let $(X, G, \leq)$ be a partially ordered complete G-metric space, and let Ω be an Ω-distance on X. Suppose that a non-decreasing self-mapping $T : X \to X$ is a generalized weak-contraction mapping, that is,*

$$\psi\bigl(\Omega\bigl(Tx, T^2x, Ty\bigr)\bigr) \leq \psi\bigl(\Omega(x, Tx, y)\bigr) - \phi\bigl(\Omega(x, Tx, y)\bigr) \quad \textit{for all } x, y \in X, \textit{ with } x \leq Tx,$$

with $\psi \in \Psi$ and $\phi \in \Phi$. Suppose also that $\inf\{\Omega(x, y, x) + \Omega(x, y, Tx) + \Omega(x, Tx, y) : x \leq Tx\} > 0$ *for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \leq Tx_0$, then T has a unique fixed point, say $u \in X$. Moreover, $\Omega(u, u, u) = 0$.*

Proof If $x_0 = Tx_0$, then the proof is finished. Suppose that $x_0 \neq Tx_0$. Since $x_0 \leq Tx_0$ and T is non-decreasing, we obtain

$$x_0 \leq Tx_0 \leq T^2x_0 \leq \cdots \leq T^{n+1}x_0 \leq \cdots .$$

Now, if for some $n \in \mathbb{N}$, $\Omega(T^nx_0, T^{n+1}x_0, T^{n+1}x_0) = 0$, then

$$\begin{aligned}\psi\bigl(\Omega\bigl(T^{n+1}x_0, T^{n+2}x_0, T^{n+2}x_0\bigr)\bigr) &\leq \psi\bigl(\Omega\bigl(T^nx_0, T^{n+1}x_0, T^{n+1}x_0\bigr)\bigr) \\ &\quad - \phi\bigl(\Omega\bigl(T^nx_0, T^{n+1}x_0, T^{n+1}x_0\bigr)\bigr),\end{aligned}$$

then $\Omega(T^{n+1}x_0, T^{n+2}x_0, T^{n+2}x_0) = 0$. Due to [(a), Definition 3], we have $\Omega(T^nx_0, T^{n+2}x_0, T^{n+2}x_0) = 0$. On the other hand, by [(c), Definition 3], we easily derive that $G(T^nx_0, T^{n+2}x_0, T^{n+2}x_0) = 0$, which completes the proof.

Consequently, throughout the proof, we suppose that $\Omega(T^n x_0, T^{n+1}x_0, T^{n+1}x_0) > 0$ for all $n \in \mathbb{N}$. Hence, we have

$$\begin{aligned}\psi\bigl(\Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr)\bigr) \leq{}& \psi\bigl(\Omega\bigl(T^{n-1}x_0, T^n x_0, T^n x_0\bigr)\bigr)\\ &- \phi\bigl(\Omega\bigl(T^{n-1}x_0, T^n x_0, T^n x_0\bigr)\bigr),\end{aligned} \tag{2.1}$$

which yields that

$$\psi\bigl(\Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr)\bigr) \leq \psi\bigl(\Omega\bigl(T^{n-1}x_0, T^n x_0, T^n x_0\bigr)\bigr).$$

As a result, we conclude that $\{\Omega(T^n x_0, T^{n+1}x_0, T^{n+1}x_0)\}$ is non-increasing. Thus, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} \Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr) = r.$$

We shall show that $r = 0$. Suppose, on the contrary, that $r > 0$. Then we have $\phi(r) > 0$. Letting $n \to \infty$ on (2.1), we obtain

$$\psi(r) \leq \psi(r) - \phi(r),$$

a contraction. Hence, we have

$$\lim_{n\to\infty} \Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr) = 0. \tag{2.2}$$

Recursively, we obtain that

$$\lim_{n\to\infty} \Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+t}x_0\bigr) = 0 \tag{2.3}$$

for every $t \in \mathbb{N}$.

Let $l \geq m \geq n$ with $m = n + k$ and $l = m + t$ $(k, t \in \mathbb{N})$. By the triangle inequality, we derive that

$$\begin{aligned}\Omega\bigl(T^n x_0, T^m x_0, T^l x_0\bigr) &\leq \Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr) + \Omega\bigl(T^{n+1}x_0, T^m x_0, T^l x_0\bigr)\\ &\leq \Omega\bigl(T^n x_0, T^{n+1}x_0, T^{n+1}x_0\bigr) + \Omega\bigl(T^{n+1}x_0, T^{n+2}x_0, T^{n+2}x_0\bigr)\\ &\quad + \cdots + \Omega\bigl(T^{m-1}x_0, T^m x_0, T^l x_0\bigr).\end{aligned}$$

Letting $n \to \infty$ in the inequality above, by keeping the limits (2.2) and (2.3), we obtain

$$\lim_{n,m,l\to\infty} \Omega\bigl(T^n x_0, T^m x_0, T^l x_0\bigr) = 0.$$

Therefore, $\{T^n x_0\}$ is a G-Cauchy sequence. Since X is G-complete, $\{T^n x_0\}$ converges to a point $u \in X$. Now, for $\varepsilon > 0$ and by the lower semi-continuity of Ω,

$$\Omega\bigl(T^n x_0, T^m x_0, u\bigr) \leq \liminf_{p\to\infty} \Omega\bigl(T^n x_0, T^m x_0, T^p x_0\bigr) \leq \varepsilon, \quad m \geq n,$$

and

$$\Omega(T^n x_0, u, T^l x_0) \leq \liminf_{p\to\infty} \Omega(T^n x_0, T^p x_0, T^l x_0) \leq \varepsilon, \quad l \geq n.$$

Assume that $u \neq Tu$. Since $T^n x_0 \preceq T^{n+1} x_0$,

$$0 < \inf\{\Omega(T^n x_0, u, T^n x_0) + \Omega(T^n x_0, u, T^{n+1} x_0) + \Omega(T^n x_0, T^{n+1} x_0, u) : n \in \mathbb{N}\} \leq 3\varepsilon,$$

a contraction. Hence, we have $u = Tu$.

We shall show that u is the unique fixed point of T. Suppose, on the contrary, that v is another fixed point of T. So, we have

$$\begin{aligned}\psi(\Omega(u,u,v)) &= \psi(\Omega(Tu, T^2u, Tv)) \\ &\leq \psi(\Omega(u, Tu, v)) - \phi(\Omega(u, Tu, v)) \\ &= \psi(\Omega(u,u,v)) - \phi(\Omega(u,u,v)) \\ &< \psi(\Omega(u,u,v)),\end{aligned}$$

a contraction. Thus, the fixed point u is unique. Now, since $u = Tu$, we have

$$\begin{aligned}\psi(\Omega(u,u,u)) &= \psi(\Omega(Tu, T^2u, Tu)) \\ &\leq \psi(\Omega(u, Tu, u)) - \phi(\Omega(u, Tu, u)) \\ &= \psi(\Omega(u,u,u)) - \phi(\Omega(u,u,u)).\end{aligned}$$

So, $\Omega(u,u,u) = 0$. □

Definition 11 Let $(X, \preceq)$ be a partially ordered space. Suppose that there exists a G-metric on X such that (X, G) is a complete G-metric space. A self-mapping $T : X \to X$ is said to be a weak-contraction mapping if it satisfies the following condition:

$$\Omega(Tx, T^2x, Ty) \leq \Omega(x, Tx, y) - \phi(\Omega(x, Tx, y)) \quad \text{for all } x, y \in X, \text{ with } x \preceq y,$$

where $\phi \in \Phi$.

Corollary 12 *Let $(X, G, \preceq)$ be a partially ordered complete G-metric space, and let Ω be an Ω-distance on X. Suppose that a non-decreasing self-mapping $T : X \to X$ is a weak-contraction mapping, that is,*

$$\Omega(Tx, T^2x, Ty) \leq \Omega(x, Tx, y) - \phi(\Omega(x, Tx, y)) \quad \text{for all } x, y \in X, \text{ with } x \preceq Tx,$$

where $\phi \in \Phi$. Suppose also that $\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \preceq Tx\} > 0$ *for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a unique fixed point, say $u \in X$. Moreover, $\Omega(u,u,u) = 0$.*

If we take $\phi(t) = kt$, where $k \in [0, 1)$, we derive Theorem 2.2 [4] as the following corollary.

Corollary 13 *Let $(X,G,\preceq)$ be a partially ordered complete G-metric space, and let Ω be an Ω-distance on X. Suppose that there exists $k \in [0,1)$ such that*

$$\Omega\bigl(Tx, T^2x, Ty\bigr) \leq k\Omega(x, Tx, y) \quad \textit{for all } x,y \in X, \textit{ with } x \preceq Tx.$$

Suppose also that $\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \preceq Tx\} > 0$ *for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \preceq Tx_0$, then T has a unique fixed point, say $u \in X$. Moreover, $\Omega(u,u,u) = 0$.*

Definition 14 Let $(X,\preceq)$ be a partially ordered space. Suppose that there exists a G-metric on X such that (X,G) is a complete G-metric space. A self-mapping $T : X \to X$ is said to be a Ćirić-type contraction mapping if it satisfies that there exists $0 \leq k < 1$ such that

$$\Omega\bigl(Tx, T^2x, Ty\bigr) \leq kM(x,x,y),$$

where

$$M(x,x,y) = \max\left\{\Omega(x, Tx, Tx), \Omega(y, Ty, Ty), \frac{1}{2}\Omega(x, Ty, Ty)\right\}$$

for all $x, y \in X$ with $x \leq y$.

Theorem 15 *Let $(X,G,\preceq)$ be a partially ordered complete G-metric space, and let Ω be an Ω-distance on X. Suppose that a non-decreasing self-mapping $T : X \longrightarrow X$ is a Ćirić-type contraction mapping.*

(i) *For every $x \in X$ and $y \in X$ with $y \neq T(y)$,*
$\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \preceq T(x)\} > 0$,

(ii) *There exists $x_0 \in X$ such that $x_0 \preceq T(x_0)$,*

then T has a fixed point u in X and $\Omega(u,u,u) = 0$.

Proof By assumption (ii), there exists $x_0 \in X$ such that $x_0 \preceq T(x_0)$. We fix $x_1 \in X$ such that $x_1 = T(x_0)$. Since T is a non-decreasing mapping, $Tx_0 \preceq Tx_1$. There exists $x_2 \in X$ such that $Tx_1 = x_2$. Recursively, we construct the sequence $\{x_n\}$ in the following way:

$$x_{n+1} = Tx_n \preceq Tx_{n+1} = x_{n+2} \quad \text{for all } n \geq 0.$$

Since T is a Ćirić-type contraction mapping, by replacing $x = x_n$ and $y = x_{n+1}$, we get that

$$\Omega(x_{n+1}, x_{n+2}, x_{n+2}) = \Omega(Tx_n, Tx_{n+1}, Tx_{n+1}) \leq kM(x_n, x_n, x_{n+1}), \tag{2.4}$$

where

$$\begin{aligned} M(x_n, x_n, x_{n+1}) &= \max\Bigl\{\Omega(x_n, Tx_n, Tx_n), \Omega(x_{n+1}, Tx_{n+1}, Tx_{n+1}), \\ &\quad \frac{1}{2}\Omega(x_n, Tx_{n+1}, Tx_{n+1})\Bigr\} \\ &= \max\Bigl\{\Omega(x_n, x_{n+1}, x_{n+1}), \Omega(x_{n+1}, x_{n+2}, x_{n+2}),\end{aligned}$$

$$\begin{aligned}
&\frac{1}{2}\Omega(x_n, x_{n+2}, x_{n+2})\bigg\} \\
&\leq \max\bigg\{\Omega(x_n, x_{n+1}, x_{n+1}), \Omega(x_{n+1}, x_{n+2}, x_{n+2}), \\
&\frac{1}{2}\big[\Omega(x_n, x_{n+1}, x_{n+1}) + \Omega(x_{n+1}, x_{n+2}, x_{n+2})\big]\bigg\} \\
&= \max\big\{\Omega(x_n, x_{n+1}, x_{n+1}), \Omega(x_{n+1}, x_{n+2}, x_{n+2})\big\}.
\end{aligned}$$

Notice that if $M(x_n, x_n, x_{n+1}) \leq \Omega(x_{n+1}, x_{n+2}, x_{n+2})$, then (2.4) yields a contradiction since $k < 1$.

Thus, $M(x_n, x_n, x_{n+1}) \leq \Omega(x_n, x_{n+1}, x_{n+1})$ and inequality (2.4) and $k < 1$ turn into

$$\Omega(x_{n+1}, x_{n+2}, x_{n+2}) \leq k\Omega(x_n, x_{n+1}, x_{n+1}). \tag{2.5}$$

Upon the discussion above, we conclude that the sequence $\{\Omega(x_n, x_{n+1}, x_{n+1})\}$ is non-increasing and bounded below. Therefore, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} \Omega(x_n, x_{n+1}, x_{n+1}) = r.$$

We shall show that $r = 0$. By a standard calculation, using inequality (2.5) and keeping $k < 1$ in mind, we obtain $\lim_{n\to\infty} \Omega(x_n, x_{n+1}, x_{n+1}) = 0$. We claim that the sequence $\{x_n\}$ is G-Cauchy. Let $l \geq m \geq n$ with $m = n + k$ and $l = m + t$ ($k, t \in \mathbb{N}$). By the triangle inequality, we derive that

$$\begin{aligned}
\Omega(x_n, x_m, x_l) &\leq \Omega(x_n, x_{n+1}, x_{n+1}) + \Omega(x_{n+1}, x_m, x_l) \\
&\leq \Omega(x_n, x_{n+1}, x_{n+1}) + \Omega(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots + \Omega(x_{m-1}, x_m, x_l).
\end{aligned} \tag{2.6}$$

On the other hand, we have

$$\begin{aligned}
\Omega(x_{m-1}, x_m, x_{m+t}) &\leq kM(x_{m-2}, x_{m-2}, x_{m+t-1}) \\
&= k\max\bigg\{\Omega(x_{m-2}, x_{m-1}, x_{m-1}), \Omega(x_{m+t-1}, x_{m+t}, x_{m+t}), \\
&\quad \frac{1}{2}\Omega(x_{m-2}, x_{m+t}, x_{m+t})\bigg\} \\
&\leq k\max\bigg\{\Omega(x_{m-2}, x_{m-1}, x_{m-1}), \Omega(x_{m+t-1}, x_{m+t}, x_{m+t}), \\
&\quad \frac{1}{2}\big[\Omega(x_{m-2}, x_{m-1}, x_{m-1}) + \Omega(x_{m-1}, x_m, x_m) \\
&\quad + \cdots + \Omega(x_{m+t-1}, x_{m+t}, x_{m+t})\big]\bigg\}.
\end{aligned} \tag{2.7}$$

By combining expressions (2.6) and (2.7), we find that

$$\begin{aligned}
&\Omega(x_n, x_m, x_l) \\
&\quad \leq \Omega(x_n, x_{n+1}, x_{n+1}) + \Omega(x_{n+1}, x_{n+2}, x_{n+2}) + \cdots + \Omega(x_{m-2}, x_{m-1}, x_{m-1})
\end{aligned}$$

$$+ k\max\Big\{\Omega(x_{m-2},x_{m-1},x_{m-1}),\Omega(x_{m+t-1},x_{m+t},x_{m+t}),\frac{1}{2}\big[\Omega(x_{m-2},x_{m-1},x_{m-1}) + \Omega(x_{m-1},x_m,x_m)+\cdots+\Omega(x_{m+t-1},x_{m+t},x_{m+t})\big]\Big\}. \tag{2.8}$$

Taking $n\to\infty$ in (2.8), we conclude that

$$\lim_{n,m,l\to\infty}\Omega(x_n,x_m,x_l)=0,$$

and hence $\{x_n\}$ is a G-Cauchy sequence due to expression (c) of Lemma 6. Since X is G-complete, $\{x_n\}$ converges to a point $u\in X$. Thus, for $\varepsilon>0$ and by the lower semi-continuity of Ω, we have

$$\Omega(x_n,x_m,u)\le\liminf_{p\to\infty}\Omega(x_n,x_m,x_p)\le\varepsilon,\quad m\ge n,$$

and

$$\Omega(x_n,u,x_l)\le\liminf_{p\to\infty}\Omega(x_n,x_p,x_l)\le\varepsilon,\quad l\ge n.$$

Assume that $u\neq Tu$. Since $x_{n+1}\le x_{n+2}$,

$$0<\inf\big\{\Omega(x_{n+1},u,x_{n+1})+\Omega(x_{n+1},u,x_{n+2})+\Omega(x_{n+1},x_{n+2},u):n\in\mathbb{N}\big\}\le 3\varepsilon$$

for every $\varepsilon>0$, that is a contraction. Therefore, we have $u=Tu$ and $\Omega(u,u,u)=0$. □

Definition 16 Let $(X,\le)$ be a partially ordered space and $f,g:X\to X$. We say that g is an f-monotone mapping if

$$x,y\in X,\quad f(x)\le f(y)\quad\Longrightarrow\quad g(x)\le g(y).$$

Theorem 17 *Let $(X,G,\le)$ be a partially ordered complete G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. Let $f:X\longrightarrow X$ and $g:f(X)\longrightarrow X$ commute, f be non-decreasing and g be an f-monotone mapping such that*:

(a) $gf(X)\subseteq f^2(X)$;

(b) $\Omega(gfx,gy,g^2x)\le kM(x,x,y)$, *where* $M(x,x,y)=\max\{\Omega(f^2x,fy,fgx),\Omega(fy,fy,gy),\Omega(f^2x,f^2x,fgx)\}$ *for all* $x,y\in X$ *with* $f(x)\le f(y)$ *and* $0\le k<1$;

(c) *for every* $x\in X$ *and* $z\in X$ *with* $f^2z\neq gfz$,

$$\inf\big\{\Omega(x,z,x)+\Omega(x,x,z)+\Omega\big(f^2x,gx,gfx\big):f^2x\le gfx\big\}>0;$$

(d) *there exists* $x_0\in f(X)$ *such that* $f(x_0)\le g(x_0)$;

then f and g have a unique common fixed point u in X and $\Omega(u,u,u)=0$.

Proof Let $x_0\in f(X)$ such that $f(x_0)\le g(x_0)$. By part (a), we can choose $x_1\in f(X)$ such that $f(x_1)=g(x_0)$. Again from part (a), we can choose $x_2\in f(X)$ such that $f(x_2)=g(x_1)$.

Continuing this process, we can construct sequences $\{x_n\}$ in $f(X)$ and $\{z_n\}$ in $f^2(X)$ such that

$$y_n = gx_n = fx_{n+1}, \tag{2.9}$$

and

$$z_n = gy_{n-1} = gfx_n = fgx_n = fy_n. \tag{2.10}$$

Since $f(x_0) \leq g(x_0)$ and $f(x_1) = g(x_0)$, we have $f(x_0) \leq f(x_1)$. Then by Definition 16, $g(x_0) \leq g(x_1)$. Continuing, we obtain

$$gx_n \leq gx_{n+1}, \quad \forall n \geq 0. \tag{2.11}$$

So, by (2.9) and (2.11), for all $t \geq 1$, $fx_n \leq fx_{n+t}$. Now, for all $s \geq 0$,

$$\begin{aligned} \Omega(z_n, z_{n+s}, z_{n+1}) &= \Omega\big(gfx_n, gx_{n+s-1}, g^2x_n\big) \\ &\leq k \max\big\{\Omega\big(f^2x_n, fy_{n+s-1}, fgx_n\big), \Omega(fy_{n+s-1}, fy_{n+s-1}, gy_{n+s-1}), \\ &\quad \Omega\big(f^2x_n, f^2x_n, fgx_n\big)\big\} \\ &= k \max\big\{\Omega(z_{n-1}, z_{n+s-1}, z_n), \Omega(z_{n+s-1}, z_{n+s-1}, z_{n+s}), \\ &\quad \Omega(z_{n-1}, z_{n-1}, z_n)\big\}. \end{aligned}$$

Then, for $s = 0$,

$$\Omega(z_n, z_n, z_{n+1}) \leq k\Omega(z_{n-1}, z_{n-1}, z_n).$$

For $s = 1$,

$$\Omega(z_n, z_{n+1}, z_{n+1}) \leq k^{1+1} \max\big\{\Omega(z_{n-1}, z_n, z_n), \Omega(z_{n-1}, z_{n-1}, z_n)\big\}.$$

For $s = 2$,

$$\Omega(z_n, z_{n+2}, z_{n+1}) \leq k^{1+2} \max\big\{\Omega(z_{n-1}, z_{n+1}, z_n), \Omega(z_{n-1}, z_{n-1}, z_n)\big\}$$

and

$$\begin{aligned} \Omega(z_{n-1}, z_{n-1}, z_n) &\leq k \max\big\{\Omega(z_{n-2}, z_{n-2}, z_{n-1}), \Omega(z_{n-2}, z_{n-2}, z_{n-1}), \Omega(z_{n-2}, z_{n-2}, z_{n-1})\big\} \\ &= k\Omega(z_{n-2}, z_{n-2}, z_{n-1}) \\ &\vdots \\ &\leq k^{n-1}\Omega(z_0, z_0, z_1). \end{aligned}$$

Therefore, for all $n \geq 1$ and $s \geq 0$,

$$\Omega(z_n, z_{n+s}, z_{n+1}) \leq k^{n+s} \max\big\{\Omega(z_{n-1}, z_{n+s-1}, z_n), \Omega(z_0, z_0, z_1)\big\}. \tag{2.12}$$

Notice that if $\Omega(z_n, z_{n+s}, z_{n+1}) \leq k^{n+s}\Omega(z_0, z_0, z_1)$, so for all $s \geq 0$, $\lim_{n\to\infty} \Omega(z_n, z_{n+s}, z_{n+1}) = 0$. If $\Omega(z_n, z_{n+s}, z_{n+1}) \leq k^{n+s}\Omega(z_{n-1}, z_{n+s-1}, z_n)$, so $\{\Omega(z_{n-1}, z_{n+s-1}, z_n)\}$ is non-increasing and bounded below. Therefore, there exists $r \geq 0$ such that

$$\lim_{n\to\infty} \Omega(z_{n-1}, z_{n+s-1}, z_n) = r.$$

We shall show that $r = 0$. By a standard calculation, using inequality (2.12) and keeping $k < 1$ in mind, we obtain $\lim_{n\to\infty} \Omega(z_{n-1}, z_{n+s-1}, z_n) = 0$. Now, for any $l \geq m \geq n$ with $m = n + k$ and $l = m + t$ $(k, t \in \mathbb{N})$, we have

$$\begin{aligned}
\Omega(z_n, z_m, z_l) &\leq \Omega(z_n, z_{n+1}, z_{n+1}) + \Omega(z_{n+1}, z_m, z_l) \\
&\leq \Omega(z_n, z_{n+1}, z_{n+1}) + \Omega(z_{n+1}, z_{n+2}, z_{n+2}) + \cdots + \Omega(z_{m-1}, z_m, z_l) \\
&\leq \Omega(z_n, z_{n+1}, z_{n+1}) + \Omega(z_{n+1}, z_{n+2}, z_{n+2}) + \cdots + \Omega(z_{m-1}, z_m, z_m) \\
&\quad + \Omega(z_m, z_{m+1}, z_{m+1}) + \cdots + \Omega(z_{m+t-1}, z_m, z_{m+t}).
\end{aligned}$$

So,

$$\lim_{n,m,l\to\infty} \Omega(z_n, z_m, z_l) = 0,$$

and consequently, by Part (3) of Lemma 6, $\{z_n\}$ is a G-Cauchy sequence. Since X is G-complete, $\{z_n\}$ converges to a point $z \in X$. Thus, for $\varepsilon > 0$ and by the lower semi-continuity of Ω, we have

$$\Omega(z_n, z_m, z) \leq \liminf_{p\to\infty} \Omega(z_n, z_m, z_p) \leq \varepsilon, \quad m \geq n,$$

and

$$\Omega(z_n, z, z_l) \leq \liminf_{p\to\infty} \Omega(z_n, z_p, z_l) \leq \varepsilon, \quad l \geq n.$$

Assume that $f^2 z \neq gfz$. Since f is non-decreasing, we obtain

$$z_n = f^2 x_{n+1} = f(fx_{n+1}) \leq f(fx_{n+2}) = gfx_{n+1} = z_{n+1},$$

then $z_n \leq z_{n+1}$. Also, for all $n \geq 1$,

$$\begin{aligned}
\Omega\bigl(f^2 z_n, gz_n, gfz_n\bigr) &= \Omega\bigl(gfz_{n-1}, gz_n, g^2 z_{n-1}\bigr) \\
&\leq k \max\bigl\{\Omega\bigl(f^2 z_{n-1}, fz_n, fgz_{n-1}\bigr), \Omega(fz_n, fz_n, gz_n), \\
&\quad \Omega\bigl(f^2 z_{n-1}, f^2 z_{n-1}, fgz_{n-1}\bigr)\bigr\} \\
&= k \max\bigl\{\Omega\bigl(gfz_{n-2}, gz_{n-1}, g^2 z_{n-2}\bigr), \Omega(fz_n, fz_n, gz_n), \\
&\quad \Omega\bigl(gfz_{n-2}, gfz_{n-2}, g^2 z_{n-2}\bigr)\bigr\} \\
&\leq k^3 \max\bigl\{\Omega\bigl(f^2 z_{n-2}, fz_{n-1}, fgz_{n-2}\bigr), \Omega(fz_{n-1}, fz_{n-1}, gz_{n-1}), \\
&\quad \Omega\bigl(f^2 z_{n-2}, f^2 z_{n-2}, fgz_{n-2}\bigr), \Omega(fz_n, fz_n, gz_n),
\end{aligned}$$

$$\begin{aligned}
&\quad \Omega\big(f^2z_{n-2},f^2z_{n-2},fgz_{n-2}\big),\Omega\big(f^2z_{n-2},f^2z_{n-2},gfz_{n-2}\big),\\
&\quad \Omega\big(f^2z_{n-2},f^2z_{n-2},fgz_{n-2}\big)\big\}\\
&= k^3\max\big\{\Omega\big(f^2z_{n-2},fz_{n-1},fgz_{n-2}\big),\Omega(fz_{n-1},fz_{n-1},gz_{n-1}),\\
&\quad \Omega\big(f^2z_{n-2},f^2z_{n-2},fgz_{n-2}\big),\Omega(fz_n,fz_n,gz_n)\big\}\\
&\vdots\\
&\le k^{2n+1}\max\big\{\Omega\big(f^2z_1,gz_1,gfz_1\big),\Omega\big(f^2z_1,f^2z_1,fgz_1\big),\\
&\quad \Omega(fz_i,fz_i,gz_i),0\le i\le n\big\}\\
&\le k^{2n+1}C,
\end{aligned}$$

where $C=\max\{\Omega(f^2z_1,gz_1,gfz_1),\Omega(f^2z_1,f^2z_1,fgz_1),\Omega(fz_i,fz_i,gz_i),0\le i\le n\}$, and consequently $\lim_{n\to\infty}\Omega(f^2z_n,gz_n,gfz_n)=0$. Therefore,

$$0<\inf\big\{\Omega(z_n,z,z_n)+\Omega(z_n,z_n,z)+\Omega\big(f^2z_n,gz_n,gfz_n\big):n\in\mathbb{N}\big\}\le 3\varepsilon$$

for every $\varepsilon>0$, that is a contraction. So, we have $f^2z=gfz$. Then, by (b),

$$\begin{aligned}
\Omega\big(gf^2z,g(gfz),g^2fz\big)&\le k\max\big\{\Omega\big(f^2fz,f(gfz),fg(fz)\big),\Omega\big(f(gfz),f(gfz),g(gfz)\big),\\
&\qquad \Omega\big(f^2(fz),f^2(fz),fg(fz)\big)\big\}.
\end{aligned}$$

So, $\Omega(gf^2z,g(gfz),g^2fz)=0$. Since X is Ω-bounded, $\Omega(gf^2z,g(gfz),g^2fz)=0<M$. Similarly, $\Omega(gf^2z,gfz,g^2fz)\le k\Omega(f^2z,f^2z,f^2z)<M$. By part (c) of Definition 3, $G(gf^2z,gfz,g^2fz)=0$. Then $g^2fz=gfz$, which implies that gfz is a fixed point for g. Now,

$$f(gfz)=gf^2z=g^2fz=gfz.$$

Then $u=gfz$ is a common fixed point of f and g.

Uniqueness. Assume that there exists $v\in X$ such that $fv=gv=v$. Hence, we have

$$\Omega(v,v,v)\le k\Omega(v,v,v),$$

and so $\Omega(v,v,v)=\Omega(u,u,u)=0$. Also, $\Omega(v,u,v)=0$. Then, by Part (c) of Definition 3, $u=v$ and $\Omega(u,u,u)=0$. □

The following corollary is a generalization of Theorem 2.1 [14].

Denote by Λ the set of all functions $\lambda:[0,+\infty)\to[0,+\infty)$ satisfying the following hypotheses:

(i) λ is a Lebesgue-integrable mapping on each compact subset of $[0,+\infty)$,
(ii) for every $\varepsilon>0$, we have $\int_0^{\varepsilon}\lambda(s)\,ds>0$,
(iii) $\|\lambda\|<1$, where $\|\lambda\|$ denotes the norm of λ.

Now, we have the following corollary.

Corollary 18 *Let $(X,G,\le)$ be a partially ordered complete G-metric space, let Ω be an Ω-distance on X, and let $T:X\to X$ be a non-decreasing self-mapping. Suppose that $\psi\in\Psi$*

and $\phi \in \Phi$ such that

$$\int_0^{\psi(\Omega(Tx,T^2x,Ty))} \lambda(s)\,ds \le \int_0^{\psi(\Omega(x,Tx,y))} \lambda(s)\,ds - \int_0^{\phi(\Omega(x,Tx,y))} \lambda(s)\,ds, \tag{2.13}$$

for all $x \le Tx$, $y \in X$, where $\lambda \in \Lambda$. Also, for every $x \in X$,

$$\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \le Tx\} > 0$$

for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \le Tx_0$, then T has a unique fixed point.

Proof Define $\gamma : [0,+\infty) \to [0,+\infty)$ by $\gamma(t) = \int_0^t \lambda(s)\,ds$, then from inequality (2.13), we have

$$\gamma(\psi(\Omega(Tx,T^2x,Ty))) \le \gamma(\psi(\Omega(x,Tx,y))) - \gamma(\phi(\Omega(x,Tx,y))),$$

which can be written as

$$\psi_1(\Omega(Tx,T^2x,Ty)) \le \psi_1(\Omega(x,Tx,y)) - \phi_1(\Omega(x,Tx,y)),$$

where $\psi_1 = \gamma \circ \psi$ and $\phi_1 = \gamma \circ \phi$. Since the functions ψ_1 and ϕ_1 satisfy the properties of ψ and ϕ, by Theorem 10, T has a unique fixed point. □

Corollary 19 *Let $(X,G,\le)$ be a partially ordered complete G-metric space, let Ω be an Ω-distance on X, and let $T : X \to X$ be a non-decreasing self-mapping. Suppose that there exists $0 \le k < 1$ such that*

$$\int_0^{\psi(\Omega(Tx,T^2x,Ty))} k\lambda(s)\,ds \le \int_0^{M(x,x,y)} \lambda(s)\,ds \tag{2.14}$$

for all $x \le Tx$, $y \in X$, where

$$M(x,x,y) = \max\left\{\Omega(x,Tx,Tx), \Omega(y,Ty,Ty), \frac{1}{2}\Omega(x,Ty,Ty)\right\}$$

and $\lambda \in \Lambda$. Also, for every $x \in X$,

$$\inf\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \le Tx\} > 0$$

for every $y \in X$ with $y \neq Ty$. If there exists $x_0 \in X$ with $x_0 \le Tx_0$, then T has a unique fixed point.

3 Application

In this section, we give an existence theorem for a solution of the following integral equations:

$$x(t) = \int_0^1 K(t,s,x(s))\,ds + g(t), \quad t \in [0,1]. \tag{3.1}$$

Let $X = C([0,1])$ be the set of all continuous functions defined on $[0,1]$. Define $G : X \times X \times X \to \mathbb{R}$ by

$$G(x,y,z) = \|x-y\| + \|y-z\| + \|z-x\|,$$

where $\|x\| = \sup\{|x(t)| : t \in [0,1]\}$. Then (X,G) is a complete G-metric space. Let $\Omega = G$. Then Ω is an Ω-distance on X. Define an ordered relation $\leq$ on X by

$$x \leq y \quad \text{iff} \quad x(t) \leq y(t), \qquad \forall t \in [0,1].$$

Then $(X,\leq)$ is a partially ordered set. Now, we prove the following result.

Theorem 20 *Suppose the following hypotheses hold*:

(1) $K : [0,1] \times [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ *and* $g : [0,1] \to \mathbb{R}$ *are continuous mappings,*

(2) K *is non-decreasing in its first coordinate and* g *is non-decreasing,*

(3) *There exists a continuous function* $G : [0,1] \times [0,1] \to [0,+\infty)$ *such that*

$$\bigl|K(t,s,u) - K(t,s,v)\bigr| \leq G(t,s)|u-v|$$

for every comparable $u, v \in \mathbb{R}^+$ *and* $s,t \in [0,1]$ *with* $\sup_{t\in[0,1]} \int_0^1 G(t,s)\,ds \leq \frac{1}{2}$,

(4) *There exist continuous, non-decreasing functions* $\phi, \psi : [0,\infty) \to (0,\infty)$ *with* $\psi^{-1}(\{0\}) = \phi^{-1}(\{0\}) = \{0\}$ *and* $\psi(r) \leq \psi(2r) - \phi(2r)$ *for all* $r \in [0,\infty)$.

Then the integral equation has a solution in $C([0,1])$.

Proof Define $Tx(t) = \int_0^1 K(t,s,x(s))\,ds + g(t)$. By hypothesis (2), we have that T is non-decreasing.

Now, if

$$\inf\bigl\{\Omega(x,y,x) + \Omega(x,y,Tx) + \Omega(x,Tx,y) : x \leq Tx\bigr\} = 0$$

for every $y \in X$ with $y \neq Ty$, then for each $n \in \mathbb{N}$, there exists $x_n \in C([0,1])$ with $x_n \leq Tx_n$ such that

$$\Omega(x_n,y,x_n) + \Omega(x_n,y,Tx_n) + \Omega(x_n,Tx_n,y) \leq \frac{1}{n}.$$

Then we have

$$\Omega(x_n,y,Tx_n) = \sup_{t\in[0,1]} |x_n - y| + \sup_{t\in[0,1]} |y - Tx_n| + \sup_{t\in[0,1]} |Tx_n - x_n| \leq \frac{1}{n}.$$

Thus,

$$\lim_{n\to\infty} x_n(t) = y(t), \qquad \lim_{n\to\infty} Tx_n(t) = y(t).$$

By the continuity of K, we have

$$\begin{aligned} y(t) = \lim_{n\to\infty} Tx_n(t) &= \int_0^1 K\Bigl(t,s,\lim_{n\to\infty} x_n(s)\Bigr)\,ds + g(t) \\ &= \int_0^1 K\bigl(t,s,y(s)\bigr)\,ds + g(t) = Ty(t), \end{aligned}$$

which is a contradiction. Therefore,

$$\inf\big\{\Omega(x,y,x)+\Omega(x,y,Tx)+\Omega(x,Tx,y):x\preceq Tx\big\}>0.$$

Now, for $x,y\in X$ with $x\preceq Tx$, we have

$$\begin{aligned}
\psi\big(\Omega\big(Tx,T^2x,Ty\big)\big) &= \psi\Big(\sup_{t\in[0,1]}\big|Tx(t)-T^2x(t)\big|+\sup_{t\in[0,1]}\big|T^2x(t)-Ty(t)\big| \\
&\quad+\sup_{t\in[0,1]}\big|Ty(t)-Tx(t)\big|\Big) \\
&\le \psi\bigg(\sup_{t\in[0,1]}\int_0^1\big|K\big(t,s,x(s)\big)-K\big(t,s,Tx(s)\big)\big|\,ds \\
&\quad+\sup_{t\in[0,1]}\int_0^1\big|K\big(t,s,Tx(s)\big)-K\big(t,s,y(s)\big)\big|\,ds \\
&\quad+\sup_{t\in[0,1]}\int_0^1\big|K\big(t,s,y(s)\big)-K\big(t,s,x(s)\big)\big|\,ds\bigg) \\
&\le \psi\bigg(\sup_{t\in[0,1]}\bigg(\int_0^1 G(t,s)\big|x(s)-Tx(s)\big|\,ds\bigg) \\
&\quad+\sup_{t\in[0,1]}\bigg(\int_0^1 G(t,s)\big|Tx(s)-y(s)\big|\,ds\bigg) \\
&\quad+\sup_{t\in[0,1]}\bigg(\int_0^1 G(t,s)\big|y(s)-x(s)\big|\,ds\bigg)\bigg) \\
&\le \psi\bigg(\sup_{t\in[0,1]}\big(\big|x(t)-Tx(t)\big|\big)\sup_{t\in[0,1]}\int_0^1 G(t,s)\,ds \\
&\quad+\sup_{t\in[0,1]}\big(\big|Tx(t)-y(t)\big|\big)\sup_{t\in[0,1]}\int_0^1 G(t,s)\,ds \\
&\quad+\sup_{t\in[0,1]}\big(\big|y(t)-x(t)\big|\big)\sup_{t\in[0,1]}\int_0^1 G(t,s)\,ds\bigg) \\
&\le \psi\bigg(\frac{1}{2}\sup_{t\in[0,1]}\big(\big|x(t)-Tx(t)\big|\big)+\frac{1}{2}\sup_{t\in[0,1]}\big(\big|Tx(t)-y(t)\big|\big) \\
&\quad+\frac{1}{2}\sup_{t\in[0,1]}\big(\big|y(t)-x(t)\big|\big)\bigg) \\
&\le \psi\bigg(\frac{1}{2}\Omega(x,Tx,y)\bigg)\le\psi\big(\Omega(x,Tx,y)\big)-\phi\big(\Omega(x,Tx,y)\big).
\end{aligned}$$

Thus, by Theorem 10, there exists a solution $u\in C[0,1]$ of integral equation (3.1). □

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this article.

Authors' contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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Acknowledgements
The authors thank anonymous reviewers for their remarkable comments, suggestions and ideas that helped to improve this paper.

Received: 19 July 2013 Accepted: 7 October 2013 Published: #PUBLICATION_DATE

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Cite this article as: Gholizadeh and Karapınar: Remarks on contractive mappings via Ω-distance. *Journal of Inequalities and Applications* #CITATION