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Generalized difference sequence spaces associated with a multiplier sequence on a real n-normed space

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Abstract

The purpose of this paper is to introduce new sequence spaces associated with a multiplier sequence by using an infinite matrix, an Orlicz function and a generalized B-difference operator on a real n-normed space. Some topological properties of these spaces are examined. We also define a new concept, which will be called $(B^{\mu}_{\Lambda})^n$-statistical A-convergence, and establish some inclusion connections between the sequence space $W(A, B^{\mu}_{\Lambda}, p, \|\cdot,\dots,\cdot\|)$ and the set of all $(B^{\mu}_{\Lambda})^n$-statistically A-convergent sequences.

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1 Introduction

Let w, l_∞, c and c_0 be the linear spaces of all, bounded, convergent and null sequences $x = (x_k)$ for all $k \in \mathbb{N}$, respectively.

Let X and Y be two subsets of w. By (X, Y), we denote the class of all matrices of A such that $A_m(x) = \sum_{k=1}^{\infty} a_{mk}x_k$ converges for each $m \in \mathbb{N}$, the set of all natural numbers, and the sequence $Ax = (A_m(x))_{m=1}^{\infty} \in Y$ for all $x \in X$.

Let $A = (a_{mk})$ be an infinite matrix of complex numbers. Then A is said to be regular if and only if it satisfies the following well-known Silverman-Toeplitz conditions:

(1) $\sup_m \sum_{k=1}^{\infty} |a_{mk}| < \infty$,
(2) $\lim_{m\to\infty} a_{mk} = 0$ for each $k \in \mathbb{N}$,
(3) $\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk} = 1$.

The idea of statistical convergence was given by Zygmund [1] in 1935. The concept of statistical convergence was introduced by Fast [2] and Schoenberg [3] independently for the real sequences. Later on, it was further investigated from a sequence point of view and linked with the summability theory by Fridy [4] and many others. The natural density of a subset E of $\mathbb{N}$ is denoted by

$$\delta(E) = \lim_{m\to\infty} \frac{1}{m}\bigl|\{k \in E : k \le m\}\bigr|,$$

where the vertical bar denotes the cardinality of the enclosed set.

Spaces of strongly summable sequences were studied by Kuttner [5], Maddox [6] and others. The class of sequences that are strongly Cesaro summable with respect to a modulus was introduced by Maddox [7] as an extension of the definition of strongly Cesaro summable sequences. Connor [8] has further extended this definition to a definition of strong A-summability with respect to a modulus, where $A = (a_{mk})$ is a non-negative regular matrix, and established some connections between strong A-summability with respect to a modulus and A-statistical convergence.

Assume now that A is a non-negative regular summability matrix. Then a sequence $x = (x_k)$ is said to be A-statistically convergent to a number L if $\delta_A(K) = \lim_{m\to\infty}\sum_{k=1}^{\infty} a_{mk} \times \chi_K(k) = 0$ or, equivalently, $\lim_{m\to\infty}\sum_{k\in K} a_{mk} = 0$ for every $\varepsilon > 0$, where $K = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ and $\chi_K(k)$ is the characteristic function of K. We denote this limit by $st_A\text{-}\lim x = L$ [9] (see also [8, 10, 11]).

For $A = C_1$, the Cesaro matrix, A-statistical convergence reduces to statistical convergence (see [2, 4]). Taking $A = I$, the identity matrix, A-statistical convergence coincides with ordinary convergence. We note that if $A = (a_{mk})$ is a regular summability matrix for which $\lim_m \max_k |a_{mk}| = 0$, then A-statistical convergence is stronger than usual convergence [10]. It should be also noted that the concept of A-statistical convergence may also be given in normed spaces [12].

The notion of difference sequence space was introduced by Kızmaz [13]. It was further generalized by Et and Çolak [14] as follows: $Z(\Delta^\mu) = \{x = (x_k) \in w : (\Delta^\mu x_k) \in Z\}$ for $Z = l_\infty, c$ and c_0, where μ is a non-negative integer, $\Delta^\mu x_k = \Delta^{\mu-1}x_k - \Delta^{\mu-1}x_{k+1}$, $\Delta^0 x_k = x_k$ for all $k \in \mathbb{N}$ or equivalent to the following binomial representation:

$$\Delta^\mu x_k = \sum_{\nu=0}^{\mu} (-1)^\nu \binom{\mu}{\nu} x_{k+\nu}.$$

These sequence spaces were generalized by Et and Başarır [15] taking $Z = l_\infty(p), c(p)$ and $c_0(p)$.

Dutta [16] introduced the following difference sequence spaces using a new difference operator: $Z(\Delta_{(\eta)}) = \{x = (x_k) \in w : \Delta_{(\eta)}x \in Z\}$ for $Z = l_\infty, c$ and c_0, where $\Delta_{(\eta)}x = (\Delta_{(\eta)}x_k) = (x_k - x_{k-\eta})$ for all $k, \eta \in \mathbb{N}$.

In [17], Dutta introduced the sequence spaces $\overline{c}(\|\cdot,\cdot\|, \Delta^\mu_{(\eta)}, p)$, $\overline{c_0}(\|\cdot,\cdot\|, \Delta^\mu_{(\eta)}, p)$, $l_\infty(\|\cdot,\cdot\|, \Delta^\mu_{(\eta)}, p)$, $m(\|\cdot,\cdot\|, \Delta^\mu_{(\eta)}, p)$ and $m_0(\|\cdot,\cdot\|, \Delta^n_{(\eta)}, p)$, where η, $\mu \in \mathbb{N}$ and $\Delta^\mu_{(\eta)}x = (\Delta^\mu_{(\eta)}x_k) = (\Delta^{\mu-1}_{(\eta)}x_k - \Delta^{\mu-1}_{(\eta)}x_{k-\eta})$ and $\Delta^0_{(\eta)}x_k = x_k$ for all $k, \eta \in \mathbb{N}$, which is equivalent to the following binomial representation:

$$\Delta^\mu_{(\eta)} x_k = \sum_{\nu=0}^{\mu} (-1)^\nu \binom{\mu}{\nu} x_{k-\eta\nu}.$$

The difference sequence spaces have been studied by several authors, [15–34]. Başar and Altay [35] introduced the generalized difference matrix $B = (b_{mk})$ for all $k, m \in \mathbb{N}$, which is a generalization of $\Delta_{(1)}$-difference operator, by

$$b_{mk} = \begin{cases} r, & k = m, \\ s, & k = m-1, \\ 0 & (k > m) \text{ or } (0 \leq k < m-1). \end{cases}$$

Başarır and Kayıkçı [36] defined the matrix $B^{\mu} = (b^{\mu}_{mk})$ which reduced the difference matrix $\Delta^{\mu}_{(1)}$ in case $r = 1$, $s = -1$. The generalized B^{μ}-difference operator is equivalent to the following binomial representation:

$$B^{\mu}x = B^{\mu}(x_k) = \sum_{\nu=0}^{\mu} \binom{\mu}{\nu} r^{\mu-\nu} s^{\nu} x_{k-\nu}.$$

Related articles can be found in [35–41].

The concept of 2-normed space was initially introduced by Gähler [42] in the mid of 1960s, while that of n-normed spaces can be found in Misiak [43]. Since then, many others have used these concepts and obtained various results; see, for instance, Gunawan [44], Gunawan and Mashadi [45], Gunawan *et al.* [46] (see also [47–54]).

2 Definitions and preliminaries

Let n be a non-negative integer and let X be a real vector space of dimension $d \geq n \geq 2$. A real-valued function $\|\cdot,\dots,\cdot\|$ on X^n satisfies the following conditions:

(1) $\|x_1,\dots,x_n\| = 0$ if and only if $x_1,\dots,x_n$ are linearly dependent,
(2) $\|x_1,\dots,x_n\|$ is invariant under permutation,
(3) $\|\alpha x_1,\dots,x_{n-1},x_n\| = |\alpha|\|x_1,\dots,x_{n-1},x_n\|$ for any $\alpha \in \mathbb{R}$,
(4) $\|x_1,\dots,x_{n-1},y+z\| \leq \|x_1,\dots,x_{n-1},y\| + \|x_1,\dots,x_{n-1},z\|$.

Then it is called an n-norm on X and the pair $(X,\|\cdot,\dots,\cdot\|)$ is called an n-normed space. A trivial example of an n-normed space is $X = \mathbb{R}^n$ equipped with the following Euclidean n-norm: $\|x_1,\dots,x_n\|_E = |\det(x_{ij})|$, where $x_i = (x_{i_1},\dots,x_{i_n}) \in \mathbb{R}^n$ for each $i = 1,\dots,n$. The standard n-norm on X, where X is a real inner product space of dimension $d \geq n$, is defined as

$$\|x_1,\dots,x_n\|_S := \begin{vmatrix} \langle x_1,x_1\rangle & \cdots & \langle x_1,x_n\rangle \\ \vdots & \ddots & \vdots \\ \langle x_n,x_1\rangle & \cdots & \langle x_n,x_n\rangle \end{vmatrix}^{\frac{1}{2}},$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product on X. If $X = \mathbb{R}^n$, then this n-norm is exactly the same as the Euclidean n-norm $\|x_1,\dots,x_n\|_E$ as mentioned earlier. Notice that for $n = 1$, the n-norm above is the usual norm $\|x_1\|_S = \langle x_1,x_1\rangle^{\frac{1}{2}}$ which gives the length of x_1, while for $n = 2$, it defines the standard 2-norm $\|x_1,x_2\|_S = (\|x_1\|_S^2.\|x_2\|_S^2 - \langle x_1,x_1\rangle^2)^{\frac{1}{2}}$ which represents the area of the parallelogram spanned by x_1 and x_2. Further, if $X = \mathbb{R}^3$, then $\|x_1,x_2,x_3\|_S = \|x_1,x_2,x_3\|_E$ represents the volume of the parallelograms spanned by x_1, x_2 and x_3. In general $\|x_1,\dots,x_n\|_S$ represents the volume of the n-dimensional parallelepiped spanned by $x_1,\dots,x_n$ in X.

A sequence (x_k) in an n-normed space $(X,\|\cdot,\dots,\cdot\|)$ is said to converge to some $L \in X$ in the n-norm if for each $\varepsilon > 0$ there exists a positive integer $n_0 = n_0(\varepsilon)$ such that $\|x_k - L, z_1,\dots,z_{n-1}\| < \varepsilon$ for all $k \geq n_0$ and for every $z_1,\dots,z_{n-1} \in X$ [45].

An Orlicz function is a function $M : [0,\infty) \to [0,\infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. It is well known that if M is a convex function, then $M(\alpha x) \leq \alpha M(x)$ with $0 < \alpha < 1$.

Let $\Lambda = (\Lambda_k)$ be a sequence of nonzero scalars. Then, for a sequence space E, the multiplier sequence space E_{Λ}, associated with the multiplier sequence Λ, is defined as

$$E_{\Lambda} = \{x = (x_k) \in w : (\Lambda_k x_k) \in E\}.$$

The following well-known inequality will be used throughout the paper. Let $p = (p_k)$ be any sequence of positive real numbers with $0 < h = \inf_k p_k \le p_k \le \sup_k p_k = H$, $D = \max\{1, 2^{H-1}\}$. Then we have, for all $a_k, b_k \in \mathbb{C}$ and for all $k \in \mathbb{N}$,

$$|a_k + b_k|^{p_k} \le D\bigl(|a_k|^{p_k} + |b_k|^{p_k}\bigr), \tag{2.1}$$

and for $a \in \mathbb{C}$, $|a|^{p_k} \le \max\{|a|^h, |a|^H\}$.

In this paper, we introduce some new sequence spaces on a real n-normed space by using an infinite matrix, an Orlicz function and a generalized B^μ_Λ-difference operator. Further, we examine some topological properties of these sequence spaces. We also introduce a new concept which will be called $(B^\mu_\Lambda)^n$-statistical A-convergence in an n-normed space.

3 Main results

In this section, we give some new sequence spaces on a real n-normed space and investigate some topological properties of these spaces. We also give some inclusion relations.

Let $A = (a_{mk})$ be an infinite matrix of non-negative real numbers, let $p = (p_k)$ be a bounded sequence of positive real numbers for all $k \in \mathbb{N}$, and let $\Lambda = (\Lambda_k)$ be a sequence of nonzero scalars. Further, let M be an Orlicz function and $(X, \|\cdot,\ldots,\cdot\|)$ be an n-normed space. We denote the space of all X-valued sequence spaces by $w(n-X)$ and $x = (x_k) \in w(n-X)$ by $x = (x_k)$ for brevity. We define the following sequence spaces for every nonzero $z_1, z_2, \ldots, z_{n-1} \in X$ and for some $\rho > 0$:

$$\begin{aligned}
&W\bigl(A, B^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|\bigr)\\
&\quad= \Biggl\{x = (x_k) : \lim_{m\to\infty} \sum_{k=1}^\infty a_{mk}\biggl[M\biggl(\biggl\|\frac{B^\mu_\Lambda x_k - L}{\rho}, z_1, \ldots, z_{n-1}\biggr\|\biggr)\biggr]^{p_k} = 0\\
&\qquad\text{for some } L \in X\Biggr\},\\
&W_0\bigl(A, B^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|\bigr)\\
&\quad= \Biggl\{x = (x_k) : \lim_{m\to\infty} \sum_{k=1}^\infty a_{mk}\biggl[M\biggl(\biggl\|\frac{B^\mu_\Lambda x_k}{\rho}, z_1, \ldots, z_{n-1}\biggr\|\biggr)\biggr]^{p_k} = 0\Biggr\},\\
&W_\infty\bigl(A, B^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|\bigr)\\
&\quad= \Biggl\{x = (x_k) : \sup_m \sum_{k=1}^\infty a_{mk}\biggl[M\biggl(\biggl\|\frac{B^\mu_\Lambda x_k}{\rho}, z_1, \ldots, z_{n-1}\biggr\|\biggr)\biggr]^{p_k} < \infty\Biggr\},
\end{aligned}$$

where and throughout the paper $B^\mu_\Lambda x_k = \sum_{v=0}^{\mu} \binom{\mu}{v} r^{\mu-v} s^v x_{k-v}\Lambda_{k-v}$ and $\mu, k \in \mathbb{N}$. If we consider some special cases of the spaces above, the following are obtained:

(1) If we take $\mu = 0$, then the spaces above are reduced to $W(A, \Lambda, M, p, \|\cdot,\ldots,\cdot\|)$, $W_0(A, \Lambda, M, p, \|\cdot,\ldots,\cdot\|)$, $W_\infty(A, \Lambda, M, p, \|\cdot,\ldots,\cdot\|)$, respectively.

(2) If we take $r = 1$, $s = -1$, then we get the spaces $W(A, \Delta^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|)$, $W_0(A, \Delta^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|)$, $W_\infty(A, \Delta^\mu_\Lambda, M, p, \|\cdot,\ldots,\cdot\|)$.

(3) If $M(x) = x$, then the spaces above are denoted by $W(A, B^\mu_\Lambda, p, \|\cdot,\ldots,\cdot\|)$, $W_0(A, B^\mu_\Lambda, p, \|\cdot,\ldots,\cdot\|)$, $W_\infty(A, B^\mu_\Lambda, p, \|\cdot,\ldots,\cdot\|)$, respectively.

(4) If $p_k = 1$ for all $k \in \mathbb{N}$ and $\Lambda = (\Lambda_k) = (1,1,1,\ldots)$, then the spaces above are reduced to the sequence spaces $W(A,B^\mu,M,\|\cdot,\ldots,\cdot\|)$, $W_0(A,B^\mu,M,\|\cdot,\ldots,\cdot\|)$, $W_\infty(A,B^\mu,M,\|\cdot,\ldots,\cdot\|)$, respectively.

(5) If $M(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then the spaces above are denoted by $W(A,B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$, $W_0(A,B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$, $W_\infty(A,B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$, respectively.

(6) If we take $A = C_1$, *i.e.*, the Cesaro matrix, then the spaces above are reduced to the spaces $W(B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_0(B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_\infty(B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$.

(7) If we take $A = (a_{mk})$ is de la Vallee Poussin mean, *i.e.*,

$$a_{mk} = \begin{cases} \frac{1}{\lambda_m}, & k \in I_m = [m-\lambda_m+1,m], \\ 0, & \text{otherwise}, \end{cases} \tag{3.1}$$

where λ_m is a non-decreasing sequence of positive numbers tending to ∞ and $\lambda_{m+1} \le \lambda_m + 1$, $\lambda_1 = 1$, then the spaces above are denoted by $W(\lambda,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_0(\lambda,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_\infty(\lambda,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$.

(8) By a lacunary sequence $\theta = (k_m)$, $m = 0,1,\ldots$, where $k_0 = 0$, we mean an increasing sequence of non-negative integers with $h_m = (k_m - k_{m-1}) \to \infty$ as $m \to \infty$. The intervals determined by θ are denoted by $I_m = (k_{m-1},k_m]$. Let

$$a_{mk} = \begin{cases} \frac{1}{h_m}, & k_{m-1} < k \le k_m, \\ 0, & \text{otherwise}. \end{cases} \tag{3.2}$$

Then we obtain the spaces $W(\theta,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_0(\theta,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$ and $W_\infty(\theta,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, respectively.

(9) If we take $A = I$, where I is an identity matrix and $p_k = 1$ for all $k \in \mathbb{N}$, then the spaces above are reduced to the sequence spaces $c(B^\mu_\Lambda,M,\|\cdot,\ldots,\cdot\|)$, $c_0(B^\mu_\Lambda,M,\|\cdot,\ldots,\cdot\|)$ and $l_\infty(B^\mu_\Lambda,M,\|\cdot,\ldots,\cdot\|)$, respectively.

(10) If we take $A = I$, where I is an identity matrix, $M(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then we denote the spaces above by the sequence spaces $c(B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$, $c_0(B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$ and $l_\infty(B^\mu_\Lambda,\|\cdot,\ldots,\cdot\|)$.

Theorem 3.1 *$W(A,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$, $W_0(A,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$ and $W_\infty(A,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$ are linear spaces.*

Proof We consider only $W(A,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$. Others can be treated similarly. Let $x,y \in W(A,B^\mu_\Lambda,M,p,\|\cdot,\ldots,\cdot\|)$ and α, β be scalars. Suppose that $x \to L_1$ and $y \to L_2$. Then there exists $|\alpha|\rho_1 + |\beta|\rho_2 > 0$ such that

$$\begin{aligned}
&\sum_{k=1}^\infty a_{mk}\left[M\left(\left\|\frac{B^\mu_\Lambda(\alpha x_k + \beta y_k) - (\alpha L_1 + \beta L_2)}{|\alpha|\rho_1 + |\beta|\rho_2}, z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k} \\
&\quad \le \sum_{k=1}^\infty a_{mk}\left[M\left(\frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2}\left\|\frac{B^\mu_\Lambda x_k - L_1}{\rho_1}, z_1,\ldots,z_{n-1}\right\|\right.\right. \\
&\qquad \left.\left. + \frac{|\beta|\rho_2}{|\alpha|\rho_1 + |\beta|\rho_2}\left\|\frac{B^\mu_\Lambda y_k - L_2}{\rho_2}, z_1,\ldots,z_{n-1B^\mu_\Lambda}\right\|\right)\right]^{p_k} \\
&\quad \le \sum_{k=1}^\infty a_{mk}\left[\frac{|\alpha|\rho_1}{|\alpha|\rho_1 + |\beta|\rho_2} M\left(\left\|\frac{B^\mu_\Lambda x_k - L_1}{\rho_1}, z_1,\ldots,z_{n-1}\right\|\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&+\frac{|\beta|\rho_2}{|\alpha|\rho_1+|\beta|\rho_2}M\left(\left\|\frac{B^{\mu}_{\Lambda}y_k-L_2}{\rho_2},z_1,\ldots,z_{n-1}\right\|\right)\Bigg]^{p_k}\\
&\leq D\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}x_k-L_1}{\rho_1},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k}\\
&\quad+D\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}y_k-L_2}{\rho_2},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k},
\end{aligned}
$$

which leads us, by taking limit as $m\to\infty$, to the fact that we get $(\alpha x+\beta y)\in W(A,B^{\mu}_{\Lambda},M,p,\|\cdot,\ldots,\cdot\|)$. □

Theorem 3.2 *For any two sequences $p=(p_k)$ and $q=(q_k)$ of positive real numbers and for any two n-norms $\|\cdot,\ldots,\cdot\|_1$, $\|\cdot,\ldots,\cdot\|_2$ on X, the following holds: $Z(A,B^{\mu}_{\Lambda},M,p,\|\cdot,\ldots,\cdot\|_1)\cap Z(A,B^{\mu}_{\Lambda},M,q,\|\cdot,\ldots,\cdot\|_2)\neq\emptyset$, where $Z=W,W_0$ and W_∞.*

Proof Since the zero element belongs to each of the above classes of sequences, thus the intersection is non-empty. □

Theorem 3.3 *Let $A=(a_{mk})$ be a non-negative matrix, and let $p=(p_k)$ be a bounded sequence of positive real numbers. Then, for any fixed $m\in\mathbb{N}$, the sequence space $W_\infty(A,B^{\mu}_{\Lambda},M,p,\|\cdot,\ldots,\cdot\|)$ is a paranormed space for every nonzero $z_1,\ldots,z_{n-1}\in X$ and for some $\rho>0$ with respect to the paranorm defined by*

$$
g_m(x)=\inf\left\{\rho^{\frac{p_m}{H}}:\left(\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}x_k}{\rho},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}}<\infty\right\}.
$$

Proof That $g_m(\theta)=0$ and $g_m(-x)=g_m(x)$ are easy to prove. So, we omit them. Let us take $x=(x_k)$ and $y=(y_k)$ in $W_\infty(A,B^{\mu}_{\Lambda},M,p,\|\cdot,\ldots,\cdot\|)$. Let

$$
\begin{aligned}
A(x)&=\left\{\rho>0:\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}x_k}{\rho},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k}<\infty\right\},\\
A(y)&=\left\{\rho>0:\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}y_k}{\rho},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k}<\infty\right\}
\end{aligned}
$$

for every nonzero $z_1,\ldots,z_{n-1}\in X$. Let $\rho_1\in A(x)$ and $\rho_2\in A(y)$, then we have

$$
\left(\sum_{k=1}^{\infty}a_{mk}\left[M\left(\left\|\frac{B^{\mu}_{\Lambda}(x_k+y_k)}{(\rho_1+\rho_2)},z_1,\ldots,z_{n-1}\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}}<\infty
$$

by using Minkowski's inequality for $p=(p_k)>1$. Thus,

$$
\begin{aligned}
g_m(x+y)&=\inf\left\{(\rho_1+\rho_2)^{\frac{p_m}{H}}:\rho_1\in A(x),\rho_2\in A(y)\right\}\\
&\leq\inf\left\{\rho_1^{\frac{p_m}{H}}:\rho_1\in A(x)\right\}+\inf\left\{\rho_2^{\frac{p_m}{H}}:\rho_2\in A(y)\right\}\\
&=g_m(x)+g_m(y).
\end{aligned}
$$

We also get $g_m(x+y) \le g_m(x) + g_m(y)$ for $0 < p_k \le 1$ by using (2.1). Hence, we complete the proof of this condition of the paranorm. Finally, we show that the scalar multiplication is continuous. Whenever $\alpha \to 0$ and x is fixed imply $g_m(\alpha x) \to 0$. Also, whenever $x \to \theta$ and α is any number imply $g_m(\alpha x) \to 0$. By using the definition of the paranorm, for every nonzero $z_1, \ldots, z_{n-1} \in X$, we have

$$g_m(\alpha x) = \inf\left\{\rho^{\frac{p_m}{H}} : \left(\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu(\alpha x_k)}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}} < \infty\right\}.$$

Then

$$g_m(\alpha x) = \inf\left\{(\alpha\varrho)^{\frac{p_m}{H}} : \left(\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\varrho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}} < \infty\right\},$$

where $\varrho = \frac{\rho}{\alpha}$. Since $|\alpha|^{p_k} \le \max\{|\alpha|^h, |\alpha|^H\}$, therefore $|\alpha|^{\frac{p_k}{H}} \le (\max\{|\alpha|^h, |\alpha|^H\})^{\frac{1}{H}}$. Then the required proof follows from the following inequality

$$\begin{aligned} g_m(\alpha x) &\le \left(\max\left\{|\alpha|^h, |\alpha|^H\right\}\right)^{\frac{1}{H}} \\ &\quad \cdot \inf\left\{\varrho^{\frac{p_m}{H}} : \left(\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\varrho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}} < \infty\right\} \\ &= \left(\max\left\{|\alpha|^h, |\alpha|^H\right\}\right)^{\frac{1}{H}} g_m(x). \end{aligned}$$

□

Theorem 3.4 *Let M, M_1, M_2 be Orlicz functions. Then the following hold*:

(1) *Let $0 < h \le p_k \le 1$. Then $Z(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|) \subseteq Z(A, B_\Lambda^\mu, M, \|\cdot, \ldots, \cdot\|)$, where $Z = W, W_0$.*

(2) *Let $1 < p_k \le H < \infty$. Then $Z(A, B_\Lambda^\mu, M, \|\cdot, \ldots, \cdot\|) \subseteq Z(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|)$, where $Z = W, W_0$.*

(3) *$W_0(A, B_\Lambda^\mu, M_1, p, \|\cdot, \ldots, \cdot\|) \cap W_0(A, B_\Lambda^\mu, M_2, p, \|\cdot, \ldots, \cdot\|) \subseteq W_0(A, B_\Lambda^\mu, M_1 + M_2, p, \|\cdot, \ldots, \cdot\|)$.*

Proof (1) We give the proof for the sequence space $W_0(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|)$ only. The other can be proved by a similar argument. Let $(x_k) \in W_0(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|)$ and $0 < h \le p_k \le 1$, then

$$\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right] \le \sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}.$$

Hence, we have the result by taking the limit as $m \to \infty$. This completes the proof.

(2) Let $1 < p_k \le H < \infty$ and $(x_k) \in W_0(A, B_\Lambda^\mu, M, \|\cdot, \ldots, \cdot\|)$. Then, for each $0 < \varepsilon < 1$, there exists a positive integer M_0 such that

$$\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right] < \varepsilon < 1$$

for all $m > M_0$. This implies that

$$\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \le \sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right].$$

Hence we have the result.

(3) Let $x = (x_k) \in W_0(A, B_\Lambda^\mu, M_1, p, \|\cdot, \ldots, \cdot\|) \cap W_0(A, B_\Lambda^\mu, M_2, p, \|\cdot, \ldots, \cdot\|)$. Then, by the following inequality, the result follows

$$\begin{aligned}
&\sum_{k=1}^{\infty} a_{mk}\left[(M_1 + M_2)\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \\
&\quad \le D\sum_{k=1}^{\infty} a_{mk}\left[M_1\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \\
&\qquad + D\sum_{k=1}^{\infty} a_{mk}\left[M_2\left(\left\|\frac{B_\Lambda^\mu x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}.
\end{aligned}$$

If we take the limit as $m \to \infty$, then we get $(x_k) \in W_0(A, B_\Lambda^\mu, M_1 + M_2, p, \|\cdot, \ldots, \cdot\|)$. This completes the proof. □

Theorem 3.5 *$Z(A, B_\Lambda^{\mu-1}, M, p, \|\cdot, \ldots, \cdot\|) \subset Z(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|)$ and the inclusion is strict for $\mu \ge 1$. In general, $Z(A, B_\Lambda^j, M, p, \|\cdot, \ldots, \cdot\|_1) \subset Z(A, B_\Lambda^\mu, M, p, \|\cdot, \ldots, \cdot\|)$ for $j = 0, 1, 2, \ldots, \mu - 1$ and the inclusions are strict, where $Z = W, W_0$ and W_∞.*

Proof We give the proof for $W_0(A, B_\Lambda^{\mu-1}, M, p, \|\cdot, \ldots, \cdot\|)$ only. The others can be proved by a similar argument. Let $x = (x_k)$ be any element in the space $W_0(A, B_\Lambda^{\mu-1}, M, p, \|\cdot, \ldots, \cdot\|)$, then there exists $\rho = |r|\rho_1 + |s|\rho_2 > 0$ such that

$$\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^{\mu-1} x_k}{\rho}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} = 0.$$

Since M is non-decreasing and convex, it follows that

$$\begin{aligned}
&\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^\mu x_k}{|r|\rho_1 + |s|\rho_2}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \\
&\quad = \sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{rB_\Lambda^{\mu-1} x_k + sB_\Lambda^{\mu-1} x_{k-1}}{|r|\rho_1 + |s|\rho_2}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \\
&\quad \le D\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^{\mu-1} x_k}{\rho_1}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k} \\
&\qquad + D\sum_{k=1}^{\infty} a_{mk}\left[M\left(\left\|\frac{B_\Lambda^{\mu-1} x_{k-1}}{\rho_2}, z_1, \ldots, z_{n-1}\right\|\right)\right]^{p_k}.
\end{aligned}$$

The result holds by taking the limit as $m \to \infty$. □

In the following example we show that the inclusion given in the theorem above is strict.

Example 3.6 Let $M(x) = x$, $p_k = 1$ for all $k \in \mathbb{N}$, $\Lambda = (\Lambda_k) = (1, 1, \ldots)$, $A = C_1$, *i.e.*, the Cesaro matrix, $r = 1$, $s = -1$, where $B^\mu_\Lambda x_k = \sum_{v=0}^{\mu} \binom{\mu}{v} r^{\mu-v} s^v x_{k-v} \Lambda_{k-v}$ for all $r, s \in \mathbb{R} - \{0\}$. Consider the sequence $x = (x_k) = (k^{\mu-1})$. Then $x = (x_k)$ belongs to $W_0(B^\mu, M, p, \|\cdot, \ldots, \cdot\|)$ but does not belong to $W_0(B^{\mu-2}, M, p, \|\cdot, \ldots, \cdot\|)$.

Theorem 3.7 *Let $A = (a_{mk})$ be a non-negative regular matrix and $p = (p_k)$ be such that $0 < h \le p_k \le H < \infty$. Then*

$$l_\infty\big(B^\mu_\Lambda, M, \|\cdot, \ldots, \cdot\|\big) \subseteq W_\infty\big(A, B^\mu_\Lambda, M, p, \|\cdot, \ldots, \cdot\|\big).$$

Proof Let $l_\infty(B^\mu_\Lambda, M, \|\cdot, \ldots, \cdot\|)$. Then there exists $T_0 > 0$ such that $[M(\|\frac{B^\mu_\Lambda x_k}{\rho}, z_1, \ldots, z_{n-1}\|)] \le T_0$ for all $k \in \mathbb{N}$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$. Since $A = (a_{mk})$ is a non-negative regular matrix, we have the following inequality by (1) of Silverman-Toeplitz conditions:

$$\sup_m \sum_{k=1}^{\infty} a_{mk} \left[M\left(\left\| \frac{B^\mu_\Lambda x_k}{\rho}, z_1, \ldots, z_{n-1} \right\| \right) \right]^{p_k} \le \max\{T_0^h, T_0^H\} \sup_m \sum_{k=1}^{\infty} a_{mk} < \infty.$$

Hence $l_\infty(B^\mu_\Lambda, M, \|\cdot, \ldots, \cdot\|) \subseteq W_\infty(A, B^\mu_\Lambda, M, p, \|\cdot, \ldots, \cdot\|)$. □

4 $(B^\mu_\Lambda)^n$-statistically *A*-convergent sequences

In this section we introduce and study a new concept of $(B^\mu_\Lambda)^n$-statistical A-convergence in an n-normed space as follows.

Definition 4.1 Let $(X, \|\cdot, \ldots, \cdot\|)$ be an n-normed space and let $A = (a_{mk})$ be a non-negative regular matrix. A real sequence $x = (x_k)$ is said to be $(B^\mu_\Lambda)^n$-statistically A-convergent to a number L if $\delta_{A(B^\mu_\Lambda)^n}(K) = \lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk}\chi_K(k) = 0$ or, equivalently, $\lim_{m\to\infty} \sum_{k\in K} a_{mk} = 0$ for each $\varepsilon > 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$, where $K = \{k \in \mathbb{N} : \|B^\mu_\Lambda x_k - L, z_1, \ldots, z_{n-1}\| \ge \varepsilon\}$ and χ_K is the characteristic function of K.

In this case, we write $(B^\mu_\Lambda)^n$*stat*-A-$\lim x = L$. $S(A(B^\mu_\Lambda)^n)$ denotes the set of all $(B^\mu_\Lambda)^n$-statistically A-convergent sequences.

If we consider some special cases of the matrix, then we have the following:

(1) If $A = C_1$, the Cesaro matrix, then the definition reduces to $(B^\mu_\Lambda)^n$-statistical convergence.
(2) If $A = (a_{mk})$ is de la Vallee Poussin mean, which is given by (3.1), then the definition reduces to $(B^\mu_\Lambda)^n$-statistical λ-convergence.
(3) If we take $A = (a_{mk})$ as in (3.2), then the definition reduces to $(B^\mu_\Lambda)^n$-statistical lacunary convergence.

Theorem 4.2 *Let $p = (p_k)$ be a sequence of non-negative bounded real numbers such that $\inf_k p_k > 0$. Then $W(A, B^\mu_\Lambda, p, \|\cdot, \ldots, \cdot\|) \subset S(A(B^\mu_\Lambda)^n)$.*

Proof Assume that $x = (x_k) \in W(A, B^\mu_\Lambda, p, \|\cdot, \ldots, \cdot\|)$. So, we have for every nonzero $z_1, \ldots, z_{n-1} \in X$

$$\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk} \left\| B^\mu_\Lambda x_k - L, z_1, \ldots, z_{n-1} \right\|^{p_k} = 0.$$

Let $\varepsilon > 0$ and $K = \{k \in \mathbb{N} : \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\| \geq \varepsilon\}$. We obtain the following:

$$\begin{aligned}
&\sum_{k=1}^{\infty} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} \\
&\quad = \sum_{k\in K} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} + \sum_{k\notin K} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} \\
&\quad \geq \min\{\varepsilon^h, \varepsilon^H\} \sum_{k\in K} a_{mk}.
\end{aligned}$$

If we take the limit as $m \to \infty$, then we get $x \in S(A(B^{\mu}_{\Lambda})^n)$. This completes the proof. □

Theorem 4.3 *Let $p = (p_k)$ be a sequence of non-negative bounded real numbers such that $\inf_k p_k > 0$. Then*

$$l_{\infty}(B^{\mu}_{\Lambda}, \|\cdot, \ldots, \cdot\|) \cap S(A(B^{\mu}_{\Lambda})^n) \subset W(A, B^{\mu}_{\Lambda}, p, \|\cdot, \ldots, \cdot\|).$$

Proof Suppose that $x = (x_k) \in l_{\infty}(B^{\mu}_{\Lambda}, \|\cdot, \ldots, \cdot\|) \cap S(A(B^{\mu}_{\Lambda})^n)$. Then there exists an integer T such that $\|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\| \leq T$ for all $k > 0$ and for every nonzero $z_1, \ldots, z_{n-1} \in X$, and $\lim_{m\to\infty} \sum_{k\in K} a_{mk} = 0$, where $K = \{k \in \mathbb{N} : \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\| \geq \varepsilon\}$. Then we can write

$$\begin{aligned}
&\sum_{k=1}^{\infty} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} \\
&\quad = \sum_{k\notin K} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} + \sum_{k\in K} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} \\
&\quad < \max\{\varepsilon^h, \varepsilon^H\} \sum_{k\notin K} a_{mk} + \max\{T^h, T^H\} \sum_{k\in K} a_{mk}.
\end{aligned}$$

Since $A = (a_{mk})$ is a non-negative regular matrix, then we have

$$\begin{aligned}
1 &= \lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk} \\
&= \lim_{m\to\infty} \sum_{k\notin K} a_{mk} + \lim_{m\to\infty} \sum_{k\in K} a_{mk}.
\end{aligned}$$

Hence, $\lim_{m\to\infty} \sum_{k\notin K} a_{mk} = 1$. Thus

$$\begin{aligned}
&\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk} \|B^{\mu}_{\Lambda}x_k - L, z_1, \ldots, z_{n-1}\|^{p_k} \\
&\quad < \varepsilon' \lim_{m\to\infty} \sum_{k\notin K} a_{mk} + T' \lim_{m\to\infty} \sum_{k\in K} a_{mk} \\
&\quad < \varepsilon',
\end{aligned}$$

where $\max\{\varepsilon^h, \varepsilon^H\} = \varepsilon'$ and $\max\{T^h, T^H\} = T'$.

Hence, $x_k \in W(A, B^{\mu}_{\Lambda}, p, \|\cdot, \ldots, \cdot\|)$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally in the preparation of this article. Both the authors read and approved the final manuscript.

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